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AN ANALYTICAL AND NUMERICAL
INVESTIGATION OF HEAT TRANSFER
IN FIRE-HEATED SOILS

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Final Report for the Research Joint Venture Agreement
INT-94968-RJVA (MSU Grants and Contracts Number 290730)

Abstract

An analytical and numerical investigation for heat transfer and temperature distribution in soil, heated from the top, is being carried out. The analytical investigation was focused on the moist zone and the numerical investigation was carried out in the dry zone. A limiting case of quasisteady state approximation was made. It was also assumed that evaporation only occurs at boiling temperature with the moist and dry zone separated by an isothermal surface. It is apparent from the study of the idealized model that the heat transfer through the dry zone is the most important phenomenon. The numerical modeling of the dry zone involves solving the two dimensional governing equations for porous media, namely, the mass, momentum, and energy equations. A finite element method based on the Galerkin technique was used. Numerical computations were performed with the objective of having a better understanding of the physics of such problems and dependencies of various parameters.

Key Words: Heat transfer, heating, soil, water vapor, fire effects.

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A. Introduction

For the past several years, a research joint venture has been prosecuted by USDA Forest Service Intermountain Research Station and the Mechanical Engineering Department of Montana State University, Bozeman. The broad objectives of this effort are to determine and to quantify the heat and mass transfer processes that occur during the heating of moist soils by wildland fire. As a first step, a review was undertaken of prior mathematical modeling efforts directed toward this end as well as a broader survey of the literature on modeling and experimentation of heat and mass transport in porous media. While no mathematical model of fire-driven heat transfer in soils has yet been widely tested and accepted, the recent model by Campbell and colleagues seems to be the most comprehensive and robust [Campbell *et al* 1993, 1994, 1995].

The broader survey included recent literature in the fields of soil science, agronomy, geohydrology, micrometeorology, and the engineering disciplines of heat transfer, fluid dynamics, and mass transport in porous media. The results of this survey are summarized in Albini *et al* [1995]. A distinctive difference was noted between the mathematical models that have arisen from the soil science field and those from the engineering and industrial process realm. In the former models, movement of liquid phase water is usually modeled using the "hydraulic conductivity" parameter, which balances frictional forces against hydrostatic pressure gradients, while vapor movement is modeled as diffusive. In models with engineering ancestry, the movement of vapor or liquid phase is usually modeled as the inertial response of a fluid to pressure and other stress tensor gradients. This means, for example, that while the former models include altered rates of percolation of liquid phase water through soil in response to temperature change, they neglect the effect of buoyancy on the movement of vapor (steam). Often models aimed at engineering applications neglect any diffusive movement of liquid or vapor phases and deal strictly with the large scale mass-average motions driven by stress tensor gradients.

The present investigation is a mix of analytical and computational analyses designed to illuminate some of the phenomenology involved in heat transfer through moist soil in a fire-heated regime including the effects of vapor transport through the soil. A limiting-case environment is constructed analytically within which numerical computations of heat and mass transport are carried out. The physical situation considered is illustrated in Fig. 1. Here, the heat from the fire heats the upper surface of a initially uniform temperature moist soil. As the heating continues, the heated surface reaches the boiling temperature of water. The liquid water then vaporizes and moves upward. As a result a dry-zone with water vapor develops and the vapor-liquid interface propagates downward into the moist (wet) zone.

The limiting case character of the model we fabricate here can be expressed as several simplifying analytical approximations:

- 1) The uniform moist soil initially fills a halfspace at constant (ambient) temperature
- 2) The upper surface is suddenly heated by constant power flux
- 3) This surface loses heat at a rate proportional to its temperature excess above ambient

- 4) The liquid phase moisture is assumed to be fixed in place until it evaporates
- 5) Evaporation occurs only at the boiling temperature
- 6) Because of (4) and (5) there exists a distinct interface between dry and moist soil zones
- 7) The interface between dry and moist soil zones is an isotherm at boiling temperature
- 8) Vapor released at the interface traverses the dry soil zone and exits at the heated surface
- 9) The dry soil / vapor zone can be described as in a quasisteady state.

At each instant the temperature field is the steady state field that satisfies the upper surface boundary condition, the isothermal lower surface boundary condition, and is consistent with the mass flux of vapor through the zone, which enters at the lower boundary at boiling temperature.

Using these idealizations, an analytical model is constructed for the rate of propagation of the interface isotherm as a function of the heat transfer rate to it, the depth of the isotherm, and the time elapsed since it began moving. In order to illustrate the behavior of this model, it is joined with a simplified analytical model for the rate of heat transfer through the evolving dry zone as a function of the thickness of the zone and the vapor flux through it. The vapor flux through the dry zone is calculable from the initial moisture content of the soil and the rate of movement of the vaporization isotherm, which relationship closes the model. By performing a series of computations in the numerical analysis for the rate of heat transfer through the dry zone for various dry zone layer thickness, with various levels of moisture vapor flux, will allow us to explore the effect of vapor movement through the soil. The analytical and numerical methods and the results obtained from these models are discussed next.

B. I. Analytical Models

As the vaporization interface moves into the moist soil, it converts the liquid phase water into vapor. The mass of vapor created per unit area per unit time is given by the product of the mass of water per unit volume and the velocity of the isotherm. The rate of heat transferred to a unit area of the interface per unit time (the power flux) must exceed the amount of heat needed to vaporize this amount of water. There must be some excess heat that passes through the interface and diffuses ahead of it, raising the moist soil temperature so that at the interface the temperature has risen to the boiling point of water. Therefore, in order to describe the rate of movement of the vaporization isotherm, one must first have a description of the temperature field in the moist soil ahead of the moving interface, because the extra amount of heat needed to raise a unit volume of the moist soil to the boiling point of water depends upon this initial temperature.

Therefore we must begin construction of the analytical model suite with a description of the temperature field in a heated halfspace. When the heated surface reaches the boiling point of water, the interface springs into existence and begins to move into the moist soil. From this instant on, the temperature in the moist soil zone is computed as the sum of two superposed fields. The first is the residual temperature field in the halfspace that begins as the temperature

distribution at the instant the heated surface reaches the boiling point, and thereafter diffuses into the halfspace without heat loss through the heated surface. The second is the temperature field created by a moving heat source of time-varying intensity. The heat source represents the excess heat flowing through the interface from the dry zone. Its rate of movement is determined by setting the sum of the two temperature fields equal to the boiling temperature of water and solving for the velocity that gives the required heat flow excess. The equation set is closed (artificially, and only for the purpose of exhibiting model behavior) by computing the steady state rate of heat transfer through a dry soil zone with the current thickness and vapor flux level, according to a simplified model.

B. I. 1. Sudden Heating of a Halfspace

The transient one-dimensional temperature field in a medium with constant thermophysical properties is governed by the diffusion equation (or Fourier equation):

$$\partial^2 T / \partial x^2 = (1/\alpha) \partial T / \partial t \quad (1)$$

where T is temperature, x is distance into the halfspace, and t is time, measured from the instant the upper surface heating begins. α is thermal diffusivity, equal to the ratio of thermal conductivity (k) to volumetric heat capacity (ρC_p). The initial condition is uniform temperature at ambient level:

$$T(x, 0) = T_0 = \text{ambient temperature,}$$

and the boundary condition at the heated surface is the so-called "radiation boundary condition":

$$\dot{Q}_{in}'' - h(T(0,t) - T_0) + k(\partial T / \partial x)(0,t) = 0 \quad (2)$$

where $\dot{Q}_{in}''$ is the incident power flux. By the artifice of defining this to be

$$\dot{Q}_{in}'' = h(T_s - T_0) \quad (3)$$

It is the ultimate temperature of the radiation-heated surface.

Expressing the temperature rise as a fraction of the temperature rise to the boiling point, $T_b - T_0$, using the dimensionless function $\Theta_{20}(x, t)$,

$$T(x,t) = T_0 + (T_b - T_0) \Theta_{20}(x,t) \quad (4)$$

the solution to the problem as stated can be written as:

$$\theta_{20} = \left(\frac{T_s - T_0}{T_b - T_0} \right) \left(\operatorname{erfc} \left(\frac{x}{\sqrt{4\alpha t}} \right) - \exp \left(\frac{hx}{k} + \left(\frac{h}{k} \right)^2 \alpha t \right) \operatorname{erfc} \left(\frac{x}{\sqrt{4\alpha t}} + \frac{h\sqrt{\alpha t}}{k} \right) \right) \quad (5)$$

where the complementary error function, erfc , is defined in terms of the error function, erf :

$$\operatorname{erfc}(\chi) = 1 - \operatorname{erf}(\chi) = 1 - \frac{2}{\sqrt{\pi}} \int_0^\chi \exp(-t^2) dt \quad (6)$$

The equation for $\theta_{20}(x, t)$ determines the time required for the heated surface to reach boiling temperature and provides the initial temperature distribution in the moist soil when the interface begins to move. The time at which the surface temperature reaches the boiling point, t_1 , is given by $\theta_{20}(0, t_1) = 1$, or

$$\left(\frac{T_b - T_0}{T_s - T_0} \right) = 1 - \exp \left(\frac{h^2 \alpha t_1}{k^2} \right) \operatorname{erfc} \left(\frac{h\sqrt{\alpha t_1}}{k} \right) \quad (7)$$

This equation defines the dimensionless parameter $\psi = h\sqrt{\alpha t_1}/k$ which characterizes the temperature field in the moist zone at the instant the interface begins to move. The value of ψ is determined by the temperature ratio on the left hand side of equation (7), allowing t_1 to be expressed in terms of parameters more readily computed or measured than the thermal conductivity of the moist soil or the effective film heat transfer coefficient. The equation defining ψ can be solved for t_1 , then simplified with h replaced using equation (3) and employing the definition of α to eliminate k to obtain, finally:

$$\left(\frac{t_1}{\alpha} \right) \left(\dot{Q}_{in}'' / \rho C_p (T_b - T_0) \right)^2 = \psi^2 \left(\frac{T_s - T_0}{T_b - T_0} \right)^2 \quad (8)$$

Note that the denominator of the squared fraction on the left hand side is the amount of heat per unit volume needed to bring the moist soil to the boiling point. The right hand side is a function only of the temperature ratio, since ψ depends only on this ratio. This relationship is graphed in Fig. 2. Note that when the temperature ratio rises above about 5, the quantity on the left hand side becomes essentially constant at a value of about 1.

The temperature history of the heated surface up to time t_1 is given by $\Theta_{20}(0, t)$, so

$$\left(\frac{T(0,t) - T_0}{T_b - T_0} \right) = \frac{1 - \exp\left(\frac{h^2 \alpha t}{k^2}\right) \operatorname{erfc}\left(\frac{h\sqrt{\alpha t}}{k}\right)}{1 - \exp(\psi^2) \operatorname{erfc}(\psi)} \quad (9)$$

This relationship is graphed in Fig. 3. Note from this figure that the temperature ratio factor in equation (5) again loses its influence as it grows larger. The dimensionless temperature histories of surfaces heated by sources having temperature excesses above boiling point of 5 and of 20 times are essentially the same.

Two dimensional quantities, t_1 , the time required for the surface to reach boiling temperature, and d_1 , where

$$d_1 = \sqrt{4\alpha t_1} \quad (10)$$

(hereafter called the "diffusion length") can be used to nondimensionalize all time and distance quantities. The dimensionless temperature rise distribution in the moist soil halfspace at the instant the interface begins to move is

$$\left(\frac{T(x) - T_0}{T_b - T_0} \right) = \theta_{20}\left(\frac{x}{d_1}, t_1\right) \quad (11)$$

where

$$\theta_{20}(\xi, t_1) = \left(\frac{T_s - T_0}{T_b - T_0} \right) (\operatorname{erfc}(\xi) - \exp(\psi^2 + 2\xi\psi) \operatorname{erfc}(\xi + \psi)) \quad (12)$$

Equation (12) is plotted in Fig. 4, again revealing the lack of influence of source temperature.

B. I. 2. Movement of the Vaporization Isotherm

Once the heated surface of the moist soil reaches the boiling point, it becomes the interface isotherm between dry and moist soil zones. Describing the movement of this isotherm in response to continued heating of the (dry zone) surface is called a "Stefan problem", in acknowledgement of its mathematical similarity to the melting ice problem originally posed by Stefan in the 1890s [Carslaw and Jaeger 1959]. Such problems are notoriously intractable and analytical solutions are the subject of continuing research [Hsieh 1995], but many computational

methods have been explored [Crank 1984]. The method we apply here might be considered an implicit "front tracking" method using Green's functions to describe the temperature field.

We describe the temperature field of the moist soil zone by the sum of two superimposed fields, each of which extends over $-\infty \leq x \leq +\infty$. Only the region of space in front of the moving interface, within the moist soil, is physically meaningful. The image space is a mathematical artifice to impose simply a "no heat loss" boundary condition at the plane $x = 0$.

At the instant the interface begins to move, we "reset the clock" and call this time zero, so from this point on, the symbol t represents time since the heated surface reached the boiling point. One component of the temperature field is the initial distribution in the moist soil zone resulting from the surface heating (Fig. 4). This initial field is replicated by reflection about the origin and allowed to diffuse throughout space thereafter. The second contribution comes from the temperature field generated by a moving, variable-strength planar heat source. This heat source represents the rate of heat transfer (power flux) into the moving interface in excess of that needed to vaporize the soil moisture at the rate the interface is moving. The initial condition for this temperature field is uniformly ambient, and it also extends over all x . It is generated by the moving interface and its mirror image propagating away in the opposite direction into the nonexistent halfspace above the surface.

B. I. 2. i. Relaxation of Initial Temperature Field

The initial temperature field described by Θ_{20} diffuses through the moist soil zone according to

$$\left(\frac{T(x,t) - T_0}{T_b - T_0} \right) = \theta_{21}(x, t) = \int_0^{\infty} \theta_{20}(x', t_1) G_1(x, x', t) dx' \quad (13)$$

where the Green's function for the boundary conditions imposed is:

$$G_1(x, t', t) = \left(\exp\left(-\frac{(x-x')^2}{4\alpha t}\right) + \exp\left(-\frac{(x+x')^2}{4\alpha t}\right) \right) / \sqrt{4\pi\alpha t} \quad (14)$$

By use of this imaged Green's function it is not necessary to replicate the initial temperature field, and integration is restricted to the range $0 \leq x \leq +\infty$. The integral in equation (13) can be evaluated numerically by Gauss-Legendre quadrature. Abscissae and weights for a symmetrical $[-1, 1]$ 100 point quadrature were generated using a popular utility subroutine [Press *et al* 1986]. Results of this numerical evaluation are presented in figures 5 and 6. To this evolving temperature field must be added that due to a moving heat source, as discussed above.

B. I. 2. ii. Temperature Field of a Moving, Variable Heat Source

In a space initially at temperature T_0 everywhere, let there be a heat source that releases power per unit volume in amount $W(x', t')$ over the plane located at x' at t' . If the volumetric heat capacity of the medium is given by ρC_p , then the temperature at location x and time $t > t'$ is given by

$$T(x, t) - T_0 = \int_0^t \int_{-\infty}^{\infty} (W(x', t')/\rho C_p) G_2(x', x; t', t) dx' dt' \quad (15)$$

where again we have chosen to use an imaged Green's function for simplicity later:

$$G_2(x, x'; t, t') = \left(\exp\left(-\frac{(x - x')^2}{4\alpha(t - t')}\right) + \exp\left(-\frac{(x + x')^2}{4\alpha(t - t')}\right) \right) / \sqrt{4\pi\alpha(t - t')} \quad (16)$$

Using the imaged Green's function allows us to specify only one source of power, namely the moving interface that propagates into the moist zone. Now if the rate of heat transfer from the dry soil zone into the interface is given by $\dot{q}_{12}''(t')$, the rate of movement of the interface in the positive x direction is $U(t')$, its location at time t' is $d(t')$, and the heat per unit volume required to evaporate the soil moisture is h_v , a little contemplation will reveal that the power release function is to be represented by:

$$W(x', t') = (\dot{q}_{12}''(t') - U(t') h_v) \delta(x' - d(t')) \quad (17)$$

Here δ is the Dirac delta function, which is zero everywhere except where its argument vanishes; at this place its integrated value is unity. It carries units of the inverse of its argument, so the units of W (power per unit volume) are consistent with the units of the rate of heat transfer, or power flux, the dimension of the quantity multiplying the delta function. Carrying out the integral over the range of x' and normalizing the temperature rise in equation (15) by $(T_b - T_0)$ provides an expression for the dimensionless temperature rise $\Theta_{22}(x, t)$ attributable to the heat "leaking through" the moving interface:

$$(T(x, t) - T_0) / (T_b - T_0) = \theta_{22}(x, t) = \int_0^t (\dot{q}_{12}''(t') - U(t') h_v) / (\rho C_p (T_b - T_0)) G_2(x, d(t'); t, t') dt' \quad (18)$$

B. I. 2. iii. Heat Transfer to the Dry / Moist Interface and Interface Velocity

The closing relationships which link the rate of heat transfer to the movement of the interface is the fact that the interface is an isotherm and that U is the rate of growth of $d(t)$:

$$\theta_{21}(d,t) + \theta_{22}(d,t) = 1 \quad (19)$$

$$\partial d / \partial t = U \quad (20)$$

The equation for the movement of d is further simplified by the quasisteady state approximation, namely that $\dot{q}_{12}''$ and U can be considered to be constants for the purpose of carrying out the integral of equation (18). If this approximation is made, then

$$d(t) - d(t') \approx U (t - t') \quad (21)$$

$$d(t) + d(t') \approx 2d(t) - U(t - t') \quad (22)$$

and the integral in (18) can be expressed in closed form:

$$\theta_{22}(d,t) = ((\dot{q}_{12}'' - U h_v) / \rho C_p U (T_b - T_0)) \cdot \Phi \left(\frac{U}{2} \sqrt{\frac{t}{\alpha}} \right) \quad (23)$$

where the function Φ is defined as:

$$\Phi(\xi) = (1 + \operatorname{erf}(\xi) - \exp(8\xi^2) \operatorname{erfc}(3\xi)) / 2 \quad (24)$$

This function has a value of 0 at $\xi = 0$ and grows asymptotically to a value of unity as $\xi \rightarrow \infty$; in the neighborhood of $\xi = 0$ it behaves as

$$\lim_{\xi \rightarrow 0} (\Phi(\xi)) = \frac{4\xi}{\sqrt{\pi}} \quad (25)$$

An equation for the velocity of the moving interface is obtained by imposing the isothermal condition of equation (19), with the result:

$$U(t) = \dot{q}_{12}''(t) / \left(h_v + \rho C_p (T_b - T_0) (1 - \Theta_{21}(d, t)) / \Phi \left(\frac{U}{2} \sqrt{\frac{t}{\alpha}} \right) \right) \quad (26)$$

Note from equation (13) that this equation can be written in a form whose physical meaning is more readily interpreted:

$$U(t) = \dot{q}_{12}''(t) / \left(h_v + \rho C_p (T_b - T(d, t)) / \Phi \left(\frac{U}{2} \sqrt{\frac{t}{\alpha}} \right) \right) \quad (27)$$

From this form and the fact that $\Phi = 1$ for large time, it is clear that the propagation speed U is that which balances the rate of heat transfer per unit area to the heat required to raise a unit volume of the moist soil at d from $T(d, t)$ to T_b and to evaporate all the moisture it contains. The function Φ in the denominator is less than unity for finite time, so more heat per unit volume than the above minimum is required to achieve the same rate of interface movement. This excess heat requirement is proportional to the product of the heat capacity per unit volume and the temperature deficiency to be overcome in order to bring the interface to the boiling point, and also depends upon the speed at which the interface is being driven.

The behavior of the velocity of the interface at $t = 0$ is difficult to extract from this formulation. The heat transfer rate in the numerator clearly goes to the limit of the absorbed power flux

$$\lim_{t \rightarrow 0} (\dot{q}_{12}''(t)) = \left(\frac{T_s - T_b}{T_s - T_0} \right) \dot{Q}_{in}'' = \dot{q}_{12}''(0) \quad (28)$$

but the behavior of the ratio

$$R_0 = (1 - \Theta_{21}(d \rightarrow 0, t \rightarrow 0)) / \lim_{t \rightarrow 0} \left(\Phi \left(\frac{U}{2} \sqrt{\frac{t}{\alpha}} \right) \right) \quad (29)$$

in the denominator of equation (25) is bothersome. If we simply set R_0 to zero, we obtain the superficially appealing result:

$$\lim_{t \rightarrow 0} (U) = \dot{q}_{12}''(0) / h_v \quad (30)$$

But, unfortunately, the limit of R_0 as t goes to zero in equation (29) is not zero. Using the first term of the small argument expansion for Φ and the leading term in the expansion of $1 - \Theta_{21}(0, t)$, which is proportional to the square root of t , gives for R_0 a constant divided by U , leading to an initial value of U that can be negative.

Returning to equation (23) and expanding Φ for small argument gives an expression for Θ_{22} that is proportional to the square root of t . Using the expansion of $\Theta_{21}(Ut, t)$ for small t in equation (19) reveals that it is satisfied identically up to the order of square root t , but without defining $U(0)$.

The conflict can be resolved by interpreting Θ_{22} in equation (23) as depending upon the implicitly determined value of U found by expanding the integrand of equation (18) about the *endpoint* of the time interval, $t' = t$ and retaining only first order terms. Thus the "initial" value of U is found at the end of a brief initial time step, and this value can be assumed to have any functional dependence upon t that is convenient for the analysis. For example, it can be assumed to be constant or to be inversely proportional to the square root of the time that has passed since interface motion began.

If the initial interface velocity is assumed to be inversely proportional to the square root of time, this implies that the location of the interface, $d(t)$, is initially proportional to the square root of t . Since $\Theta_{22}(d, t)$ does not depend explicitly upon $d(t)$, the value of $1 - \Theta_{21}(d, t)$ is computed for $d = 2Ut$, as required by the assumed square root dependence of d upon t . So equation (19) is solved by picking a small initial value for d and varying t so as to satisfy the equation for Θ_{22} with $U = d / 2t$. This leaves the actual initial value of U undefined.

Numerical solutions generated to date under the assumption that the initial interface velocity is inversely proportional to the square root of time extrapolate smoothly to an end point $U(0)$ not far from $U = 0$, and exhibit much smaller discontinuities in the slope of $U(t)$ at the end of the initial time step than do solutions generated under the assumption that the initial value of U is constant near $t = 0$. It should also be noted that an initial square root dependence of d upon t is consistent with experimental data.

Using this formulation one need only specify $\dot{q}_{12}''(t)$ or relate this quantity parametrically to d , U , and other known parameters to close the model for prediction of the temperature field and the location of the dry / moist interface as a function of time. To specify $\dot{q}_{12}''(t)$ we shall rely upon the predictions of the general purpose computer simulation of heat and mass transport through a porous medium (the dry soil zone through which flows the evaporated moisture). But in order to exhibit the response of the analytical portion of this hybrid algorithm, we use an oversimplified analytical model for the heat transfer through the evolving dry soil zone, as explained in the next section.

B. II. A Simple Analytical Model for Heat Transfer Through the Dry Soil Zone

We construct here an admittedly oversimplified model for the transient temperature and heat transfer fields in the dry soil zone. The simplifying assumptions introduced to accomplish this are:

- 1) The vapor and the soil matrix have the same temperature everywhere
- 2) The only mode of heat transfer downward is thermal conduction through the soil matrix
- 3) The only mode of heat transfer upward is by water vapor moving through the soil matrix
- 4) The volumetric heat capacity of the medium is that of the dry soil alone
- 5) All thermophysical properties are treated as constants
- 6) Kinetic and potential energy of the vapor are negligible
- 7) Quasisteady conditions apply for vapor mass flux, heat flux, and the temperature field
- 8) Vapor mass flux is determined by the rate of movement of the isothermal interface

Invoking the first three approximations permits specification of the rate of heat transfer through the dry soil zone as a simple expression involving two terms:

$$\dot{q}'' = -k_m \partial T / \partial x - \dot{m}'' C_{pv} (T - T_b) \quad (31)$$

where $\dot{q}''$ is the rate of heat transfer per unit area or power flux (positive downward), k_m is the dry soil matrix thermal conductivity, $\dot{m}''$ is the vapor mass flux (positive upward), C_{pv} the specific heat capacity of the water vapor, T the local temperature, and T_b the boiling point, which is taken for convenience as the reference temperature for vapor enthalpy. The distance x is measured from the upper, heated, surface and has the value d at the lower boundary of the dry soil zone.

Invoking the next three assumptions simplifies the expression for the conservation of energy to:

$$\rho_m C_{pm} \partial T / \partial t = - \partial \dot{q}'' / \partial x \quad (32)$$

where t is time and $\rho_m C_{pm}$ is the volumetric heat capacity of the dry soil matrix.

The boundary and initial conditions for the evolution of the heat transfer and temperature fields are

$$\dot{q}''(0, t) - \dot{Q}_{in} \left(\frac{T_s - T(0, t)}{T_s - T_0} \right) = 0 \quad (33)$$

$$T(d, t) = T_b = T(x, 0) \quad (34)$$

and the equation for the evolution of the temperature field is

$$k_m \partial^2 T / \partial x^2 + \dot{m}'' C_{pv} \partial T / \partial x = \rho_m C_{pm} \partial T / \partial t \quad (35)$$

Invoking the quasisteady approximation dismisses the time dependence of $T(x, t)$ and permits the simple solution for $T(x)$:

$$T(x) = T_b + (T_s - T_b) \Theta_1(x; d; \phi; L) \quad (36)$$

$$\Theta_1 = \frac{\phi(\exp(-x/L) - \exp(-d/L))}{\phi - (\phi - 1) \exp(-d/L)} \quad (37)$$

where parameters L and ϕ are defined as

$$L = k_m / (\dot{m}'' C_{pv}); \quad \phi = hL / k_m \quad (38)$$

From equations (31), (36), (37), and (3) the heat transfer rate to the interface can be expressed in the form

$$\dot{q}_{12}''(t) / \dot{Q}_{in}'' = \left(\frac{T_s - T_b}{T_s - T_0} \right) \exp(-d/L) / [\phi - (\phi - 1) \exp(-d/L)] \quad (39)$$

where and parameter ϕ can be written in the form

$$\phi = \dot{Q}_{in}'' / \dot{m}'' C_{pv} (T_s - T_0) \quad (40)$$

Note that the vapor mass flux is proportional to the rate of movement of the interface:

$$\dot{m}'' = U h_v / h_{fg} \quad (41)$$

where h_{fg} is the latent heat (enthalpy) of evaporation of water. From this form we observe that the parameter ϕ varies inversely with U . In order to simplify the algorithm that evaluates equation (39) we approximate the initial value of ϕ by using the incorrect but, of necessity not far wrong, initial velocity of the interface given by equation (30). This renders the initial value

of ϕ a simple expression depending only on the source temperature excess above ambient. Using equations (39), (41), and (30) in (40) reduces the initial value of ϕ to the form:

$$\phi(t = 0) = \phi_0 = h_{fg} / (C_{pv}(T_s - T_b)) \quad (42)$$

For SI units of measure, the ratio h_{fg} / C_{pv} is approximately 1100 K. Using this starting value, and revising the parameter with each time step taken using

$$\phi(t) = \phi_0 U_0 / U(t) \quad (43)$$

where U_0 is the (admittedly incorrect) initial velocity of the interface from equation (30). The alternative to this procedure is to use not the actual initial value of U but the first estimate of it found as described in the previous section, since $U(0)$ remains undefined. After the first time step, the value of ϕ is revised to consistency with this first estimate of U and its value for the first time actually influences the rate of heat transfer into the interface, since $d(0) = 0$ in equation (39). So the net effect of this starting value for ϕ is to desynchronize its evaluation by at most one time step.

B. III. Dimensional and Dimensionless Quantities Used

Care has been taken in the development of the analytical expressions above to try to employ only quantities that can be readily estimated or directly observed. Here we summarize the suite of dimensional and dimensionless parameters required to execute the algorithms embodying the analyses presented above.

The time domain is measured in terms of t_1 , the time required for the heated surface of the moist soil to rise to the boiling temperature of water. This time is found to be proportional to the product of the thermal diffusivity of the moist soil and the square of the ratio of the heat required to raise a unit volume of the moist soil to the boiling temperature to the incident power flux that heats the surface. The proportionality factor depends upon the source temperature, boiling temperature, and ambient temperature -- see equations (7) and (8) -- but only weakly, as illustrated in Fig. 2.

While the surface temperature is rising to the boiling point, heat diffuses into the moist soil, resulting in a temperature profile that decays roughly exponentially with distance from the surface. At a depth of one "diffusion length" -- see equation (10) -- the temperature rise has decreased to 0.1 times the rise at the surface, where it has just reached boiling point. See equations (11) and (12) and Fig. 4.

To characterize the heat source and to model the heated surface boundary condition, we have introduced the "source temperature" T_s . In the heating rate model used here, this temperature is the ultimate value that the heated surface will achieve. Ratios of temperature rises occur in

virtually all facets of this analytical development -- see equations (5), (8), (12), and (39) -- requiring the specification of a minimum of two quantities:

- 1) The difference between boiling temperature and ambient temperature, $(T_b - T_o)$
- 2) The ratio of the source temperature excess above ambient to this difference, $(T_s - T_o)/(T_b - T_o)$

The moisture content of the moist soil plays an important role in governing the flow of heat and the movement of the dry / moist interface, although this fact is somewhat obscured by the development presented here. Equation (23) for the dimensionless temperature field Θ_{22} reveals the requirement to specify this quantity in terms of the ratio of the heat required to raise a unit volume of moist soil from ambient temperature to the boiling point to the heat required to vaporize the moisture in the unit volume, starting from boiling temperature. This ratio, μ , depends upon the moisture content of the soil since the numerator includes the heat required to raise not only the water in a unit volume of moist soil but also the dry soil matrix to the boiling point, while the denominator depends solely upon the mass of water contained in that unit volume:

$$\mu = \rho C_p (T_b - T_o) / h_v \quad (44)$$

It may seem odd, but the ratio of the heat transfer rate to the interface $\dot{q}_{12}$ to the numerator of μ can be related to the incident heat flux on the heated surface -- equation (39) -- and hence to t_1 and the temperature ratio in 2) above -- see equation (8) -- so the other term in equation (23) does not require another parameter.

Finally, the characteristic length L defined in equation (38) depends upon the thermal conductivity of the dry soil matrix, k_m . In order to avoid the need to specify this elusive dimensional quantity, the algorithm uses the ratio of the thermal conductivity of the dry soil matrix to that of moist soil, k_m / k . Using the expressions for ψ and d_1 -- see equations (7) and (10) -- and the definition of L , one can readily derive the expression:

$$\frac{L}{d_1} = \left(\frac{\phi}{2\psi} \right) \left(\frac{k_m}{k} \right) \quad (45)$$

This is the equation used to compute the normalized values of L used in the computer algorithms. As discussed above -- equation (7) *et. seq.* -- the parameter ψ depends only upon the temperature ratio described in 2) above.

B. IV. Example Calculations and Asymptotic Limits

B. IV. 1. Example Calculations

To illustrate the performance of the analytical model outlined above, example calculations were carried out. The examples were intended to represent a sandy soil with about 35 % void volume, for three moisture conditions:

- 1) Fully saturated $\mu = 0.5$, $k / k_m = 0.5$. As noted in private communication with Prof. Gaylen Campbell, the ratio k/k_m in this case should be ≈ 0.1 , not 0.5, and the ratio is probably too large in the second instance as well.
- 2) 50% of saturation $\mu = 1.0$, $k / k_m = 0.5$.
- 3) Quite dry $\mu = 5.0$, $k / k_m = 1.0$.

For each case calculated, the same three temperatures were used:

Ambient temperature	30 C
Boiling temperature	100 C
Source temperature	590 C

To calculate the temperature profiles at various times, the expression for Θ_{21} in equation (13) was evaluated by quadrature and added to the result of a simple stepwise numerical integration of the expression for Θ_{22} in equation (18).

Results are presented graphically in terms of dimensionless temperature rises as a function of time (normalized by t_1 , the time required for the surface to reach the boiling point) and / or distance from the heated soil surface (normalized by d_1 , the diffusion length at time t_1) and the location of the dry / moist interface normalized by d_1 and the velocity of the interface, normalized by d_1 / t_1 .

Figs. 7 - 9 show the history of the heated surface temperature rise beyond the time required to reach boiling temperature. Fig. 10 shows the variation of the rate of heat transfer to the interface with time, Fig. 11 shows the location of the interface as a function of time, and figure 12 shows its velocity. Figs. 13 - 15 show temperature distributions in the three media at times of 2 and 10 times that required for the surface to reach the boiling point.

The variations of the quantities over time and space, and their sensitivities to the moisture conditions are heuristically satisfying.

B. IV. 2. Asymptotic Limit

The behavior of the system as time increases without bound we describe as its asymptotic limit. This limit is most readily revealed by examining the behavior of the temperature fields $\Theta_{21}(x,t)$ and $\Theta_{22}(x,t)$ as t increases without limit. From Fig. 4 we note that the domain of $\Theta_{20}(x',t_1)$ in

in the integrand of equation (13) is restricted roughly to $0 \leq x' \leq d_1$. So the behavior of the integral becomes insensitive to the shape of Θ_{20} as t becomes much larger than t_1 . Indeed on these considerations alone we deduce that the asymptotic behavior of Θ_{21} is:

$$\lim_{t \rightarrow \infty} \Theta_{21}(x, t) \propto \sqrt{t_1/t} \exp(-(x/d_1)^2 t_1/t) \quad (46)$$

Therefore if $x/d_1 \gg \sqrt{(t/t_1)}$ then Θ_{21} is exponentially small, while if the opposite inequality obtains, then Θ_{21} is small as $\sqrt{(t/t_1)}$, which is also small in the limit considered. Thus in either domain of x it should be permissible to dismiss Θ_{21} .

The asymptotic behavior of Θ_{22} is readily derived by transformation to a coordinate system fixed to the steadily advancing interface being driven by a constant power flux and seeking a steady state solution in the moving coordinate system. Imposing the condition that the temperature rise far ahead of the advancing interface is nil, and that the temperature at the interface location is equal to the boiling temperature provides the limiting temperature profile:

$$\Theta_{22}(x, t) = \exp(-(U_\infty / \alpha)(x - U_\infty t)) \quad (47)$$

This form can also be recovered from the limiting form of the integral expression for Θ_{22} for large time and constant interface velocity in equation (18). This method also provides the value of U_∞ from the isotherm condition:

$$\lim_{t \rightarrow \infty} (\Theta_{22}(d, t)) = (\dot{q}_{12}'' - U_\infty h_v) / \rho C_p U_\infty (T_b - t_0) \quad (48)$$

$$U_\infty = \dot{q}_{12}'' / (\rho C_p (T_b - T_0) + h_v) \quad (49)$$

The asymptotic behavior of the temperature and heat transfer fields in the simplified model for the dry soil zone is the same as that described above, being the quasisteady state solution described by equations (36) - (43). Using these equations to link the incident power flux to $\dot{q}_{12}''$ and equation (49) to establish $U(t)$ provides a simple closed set of equations. The solution for t less than a few multiples of t_1 is obviously meaningless, but the behavior for t/t_1 sufficiently large should eventually become as valid as is this entire model if the asymptotic limit is achieved.

Numerical results created using the asymptotic form of the model -- equations (36) - (43) and (49) -- are given in the Figs. 16 - 24. Figs. 16 - 18 give the asymptotic temperature history of

the heated upper surface for the three moisture conditions used in creating Figs. 7 - 9. These figures are all similar in character.

Fig. 19 shows the asymptotic solution for the rate of heat transfer to the interface versus time for the same three moisture cases. The asymptotic formulation yields heat transfer rates for the same times that are much lower than those predicted by the more complete model (Fig. 10), presumably because the depth of the dry / moist interface is predicted to be greater by the asymptotic model than by the more complete model at the same time (compare Figs. 11 and 20). The rate of movement of the interface is *faster* in the asymptotic model than in the more complete model for the same time, as revealed by comparison of Figs. 21 and 12.

So while the heat transfer rates are smaller, the movement rates of the interface are larger in the asymptotic model. The only explanation for this apparently irreconcilable behavior lies in the fact that in the more complete model, a changing fraction of the heat transferred to the interface passes through it and heats the moist soil zone ahead of it, bringing the sum of the two transient temperature fields to the boiling point at the interface location. In the asymptotic model, this process has presumably come to equilibrium, and there is a moving temperature field which has a boiling point isotherm at the interface location. To achieve this condition, excess heat must have passed through the interface for some time, since the field Θ_{22} is initially zero everywhere and the energy density in the Θ_{21} field declines uniformly with time.

Comparing the temperature profiles predicted by the two forms of the model (Fig. 13 vs 22, 14 vs 23, and 15 vs 24) shows a very extensive heated zone ahead of the advancing interface in the asymptotic model while the more complete model shows a spatially quite confined zone of excess temperature ahead of the interface. So the energy required to create the extensive region of heated moist soil ahead of the interface has not "leaked through" the moving interface in the more complete model at the time the latest of these snapshot profiles was made, at $t / t_1 = 10$. In the asymptotic model, this precursor temperature field is *assumed to exist at the outset*. This is why the interface moves faster with a lower heat transfer rate in the asymptotic model.

In fact, for the conditions investigated here, the asymptotic condition may never be achieved. If the rate of decay with time of the heat transfer rate to the interface is sufficiently rapid, the creation of the "asymptotic" heated zone ahead of the moving interface may be a thermodynamic impossibility for the rate of movement of the interface given by the more complete model. For this reason, the asymptotic model should not be used without testing convergence of the complete model also.

C. Numerical Modeling

C. I. Governing Equations

As mentioned earlier, a simplified schematic representation of the problem is shown in Fig. 1. In the previous sections the heat transfer mechanism in the moist zone is analyzed by using a simplified analytical method. This section discusses the numerical method and results obtained

from the numerical analysis of the dry zone. The numerical procedure involves solving the two dimensional governing equations that control the heat transfer and fluid flow in the medium. These are continuity, momentum, and energy equations. The equations for porous medium were used for this purpose. These equations are:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (50)$$

(51)

$$\left(\frac{\mu}{\kappa} \right) u_i = - \frac{\partial P}{\partial x_j} \delta_{ij} + \rho g_i$$

$$(\rho C)_e \frac{\partial T}{\partial t} + (\rho C) u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left(k_e \frac{\partial T}{\partial x_j} \right) \quad (52)$$

In equations (50)-(52), u_i is the volume averaged velocity component (called Darcy velocity) in the x_i coordinate direction, t is the time, T is the temperature, P is the volume averaged pressure, κ is the permeability (an empirical constant for the porous medium), μ is the fluid viscosity, ρ is the density, C is the specific heat, k is the thermal conductivity, δ_{ij} is the unit tensor and g_i is the gravitational vector. The subscript e indicates an effective property based on porosity (liquid volume fraction) weighted averages of matrix and fluid properties.

A finite element method based on Galerkin technique is used for the numerical method. The purpose of the numerical analysis is to investigate the importance of the vapor transport in the heat transfer through the dry zone. A series of computations is performed for different thickness of dry zone with various levels of moisture vapor flux and power flux. The goal is to have a better understanding of the mechanism that controls the heat transfer through soil under such condition. The results will also show the dependencies of various parameters on the heat transfer rate and temperature distribution of the medium.

C. II. Problem Formulation

The dry zone is created from the moist zone, which is initially at a uniform temperature. The upper surface of the moist zone is suddenly heated by a constant power flux ($\dot{Q}''_{in}$) radiant heater. The upper surface loses part of the radiant heat back to the ambient at a rate proportional to its temperature difference from the ambient. The rest of the heat ($\dot{q}''_{01}$) is being transmitted in the soil. As the heating from the radiant heater continues, the heat traverses

through the dry zone and is eventually transmitted into the moist zone through the vapor-liquid interface. If the value of this transmitted heat ($\dot{q}_{12}''$) is above a certain value (determined on the basis of moist soil saturation and latent heat of vaporization), then liquid water at the interface is vaporized. The water vapor then moves upward and the liquid-vapor interface propagates downward. As a result, a dry zone with water vapor is created.

The water vapor produced at the interface is called vapor flux $\dot{m}''$ and is expressed as:

$$\dot{m}'' = \rho_v \epsilon V \quad (53)$$

where,

ρ_v = density of water vapor at saturation.

ϵ = porosity (void volume fraction) of the soil

V = vapor velocity within the pores.

The water vapor flows upward through the dry zone and opposes the direction of heat flow from the top surface. The water vapor eventually exits from the dry zone at the heated surface. The quasisteady state approximation of the numerical technique allows us to assume that the vapor-liquid interface is fixed in space. As such the vapor flux is applied as a boundary condition at the lower surface. To properly understand the effect of the vapor flux on the results of the numerical computations, the method of applying the vapor flux boundary condition and its implication in the numerical code need some explanation. It is discussed next.

C. III. Vapor Mass Flux

In the present analysis, it is assumed that the vapor-liquid interface is an isotherm at boiling temperature and the dry zone is described as quasisteady state. With these assumptions, the numerical computations were performed for various depths of the dry zone (d). The vapor mass flux, as described by equation (1), can then be translated as a vapor velocity (V), because the vapor density and the soil porosity are constants. This vapor velocity is then applied as the boundary condition at the bottom with the direction of vapor motion being upward.

Thus, in the numerical computation, the vapor mass flux is applied as vapor velocity boundary condition. By applying this boundary condition, the effect of buoyancy force is considered in an indirect way. Here, liquid water is vaporized and moves upward due to the buoyancy effect. This upward motion of the water vapor is represented as a velocity boundary condition at the liquid-vapor interface. So, the numerical computation is treated as a forced convection problem, although the velocity applied as boundary condition is in fact created due to the buoyancy effect of the water vapor at the interface.

C. IV. Code Validation

The governing equations that control the heat transfer and fluid flow for the present problem were solved by using the finite element code NACHOS II, developed by Gartling (1987, a b).

It is a general purpose code designed to solve the two-dimensional mass, momentum, and energy equations for incompressible flow problems. This code has been well tested with the solutions of many benchmark problems in fluids and in porous media. Results of some of these testings are documented by Gartling (1977), Gartling (1980), and Gartling and Hickok (1985).

NACHOS II code has also been extensively used by the MSU team for various heat and fluid flow problems resulting in several archival journal publications. To prove proper implementation of the code for the present project, some results from Gartling and Hickok (1985) involving heat transfer in porous medium were recreated by the MSU team. These results were shown to the IFSL personnel at a meeting in Missoula and subsequently documented by Amin and Albin (1995).

C. V. Numerical Results and Discussion

As mentioned earlier, the purpose of the numerical analysis was to investigate the importance of vapor transport on the heat transfer through dry soil where the overall direction of heat flow is opposite to the direction of vapor transport. A series of computations with various levels of vapor influx through different types of soils was performed. Different values of soil depth with various levels of radiant heat source intensity were used. A total of 60 cases were run. Representative results from these computations are discussed here.

Two different types of soil, namely Quincy Sand and Palouse B Clay were used in this investigation. Radiant heat source intensities of 39 kw/m^2 , 54 kw/m^2 , and 70 kw/m^2 were used. The depth of the medium was varied from 0.5 cm to 2.5 cm. Representative values of vapor mass flux were chosen so that it covers the range of values that were extracted from the experimental data of Campbell *et al.* (1992). As discussed earlier, in the present investigation, the vapor mass flux is expressed in terms of vapor velocity (V). This vapor velocity is then used as one of the boundary conditions. The numerical computation continued until steady state solution was obtained. This is done because of the quasisteady state approximation of the analysis.

Before presenting the final results, it is appropriate to briefly discuss the intermediate stages. That is, the stages before the steady state solution is obtained. This would enhance the understanding of the physics of the current problem. Fig. 25 shows the temperature distribution (isotherms) of a 2 cm deep Quincy Sand medium at different times. Fig. 25 (a) shows the isotherms at an early stage and Fig. 25 (c) represents the steady state temperature distribution. An intermediate stage is shown in Fig. 25 (b).

To explain Fig. 25 and subsequent results, it is worth repeating the boundary conditions. Here the medium is heated at the top surface by a radiant heat source. This heat is transmitted through the medium and heats the liquid water at the vapor-liquid interface. The liquid evaporates when the boiling temperature is reached. Water vapor thus produced moves upward due to the buoyancy force. This vapor influx is applied as a vapor velocity boundary condition at the bottom surface with the direction of flow being upward. To represent that the medium

is infinitely long in the horizontal direction, the vertical walls of the computational domain are treated as adiabatic. The numerical computation continued until steady state condition is reached.

From Fig. 25 (a) it can be seen that at an earlier time, the effect of heating at the top is confined mainly in the upper region. The rest of the medium is still at a uniform temperature. As time passes, the heat penetrates deeper into the medium and the isotherms move downward. Finally it reaches steady state, and is shown in Fig. 25 (c). It should be noted here that each horizontal line inside the domain represents a line of constant temperature. The highest temperature being at the top and the lowest temperature at the bottom. Fig. 25 shows that even after steady state condition is reached, the isotherms are not evenly distributed in the region. That is, they are packed more near the top and relatively less concentrated toward the bottom. This is because of the continuous vapor flow from the bottom. This effect will be discussed more when the case without any vapor flow is introduced.

Fig. 26 shows the heat flux vectors at three different times with Fig. 26(c) being the steady state condition. From the relative lengths of these vectors it can be seen that most of the heat transfer occurs near the upper zone during the earlier time (Fig. 26a). As time passes, heat transfer takes place throughout the medium. The heat flux vectors also indicate that even after steady state is reached (Fig. 26c), the magnitude of heat flux is higher near the upper region in comparison to the magnitude near the bottom. That is, heat flux reduces with the increase in depth. This is because of the previously described effect of the vapor flow that opposes the overall direction of heat flow. These observations are consistent with the nature of the isotherms shown in Fig. 25.

Fig. 27 shows the temperature histories from initial state to steady state condition. Temperature profiles at different depths are shown with the temperature of the top and bottom surfaces being the first and last curves respectively. It can be seen that temperatures at different depths start from the same initial value and then gradually increase. The increase in temperature continues until the final steady state condition is reached. The last curve represents the isothermal vapor-liquid interface. To have a better understanding of the effect of vapor flow through the medium, Fig. 28 is introduced. In this case the numerical computation is performed without any vapor flux at the bottom surface. Steady state condition isotherms and heat flux vectors are shown in Figs. 28(a) and 28(b) respectively. It can be seen that as there is no mass influx from the bottom, at steady state, the isotherms are evenly distributed throughout the medium. As such, the magnitude of heat flux is same at different depths and can be seen in Fig. 28(b).

By comparing the isotherms and heat flux vectors of Figs. 28(a) and 28(b) with those of Figs. 25(c) and 26(c) respectively, the effect of the vapor influx on the temperature field in the heated medium is clearly visible. It can be seen that without any mass influx at the bottom, at steady state, the magnitude of heat flux coming from top surface remains the same when it exits the dry zone through the bottom surface. In this case, it becomes a conduction heat transfer problem without the presence of any advection/convection. Effects of various parameters on the temperature distribution and heat transfer rate are discussed next.

The parameters that are varied in this study are the depth of the medium (d) representing the location of the vapor-liquid interface, vapor velocity (V) representing the vapor mass influx, radiation heat source intensity ($\dot{Q}''_{in}$) and the type of soil. Effects of changing these parameters were studied on the temperature distribution and the heat transfer rates in the medium. Fig. 29 shows the effect of the depth of the medium on the temperature of the top surface (T_{top}). It can be seen that with the increase of the depth, the temperature at the top surface increases. This is because as the depth increases, the heat transfer resistance through the soil also increases. So, the heat transfer rate decreases, resulting in the increase of the top surface temperature.

Fig. 29 also shows that the increase of T_{top} with the depth is more at lower values of vapor flux. This is because (as will be shown later) that at lower values of vapor flux, the decrease of heat transfer at the top surface ($\dot{q}_{01}''$) is also higher. On the other hand, for higher values of vapor flux, the increase in T_{top} is smaller. Eventually it approaches an asymptotic value. This shows that after a certain value of the depth d , the temperature at the upper boundary reaches a constant value at higher rates of vapor influx from the bottom.

To show the effect of soil type on the upper surface temperature (T_{top}), Fig. 30 is introduced. This figure shows that the top surface temperature is higher for sand in comparison with clay. This is because sand has lower thermal conductivity than clay. Which in turn means that sand has higher thermal resistance. Hence (as will be shown later), the heat transfer at the top ($\dot{q}_{01}''$) is lower for sand. This causes the increased surface temperature for soils with lower thermal conductivity. Fig. 31 shows the effect of the vapor velocity on the top surface temperature for Quincy Sand. It can be seen that with the increase in the velocity, the temperature decreases. The extreme case of $V = 0$ is also included here. It should be noted here that when $V = 0$, it means there is no vapor mass influx at vapor-liquid interface. This can only occur when the heat flux at the interface ($\dot{q}_{12}''$) is below a certain value. More about this minimum value of $\dot{q}_{12}''$ is discussed later.

The variation of heat flux at the top surface ($\dot{q}_{01}''$) with the depth of the medium is shown in Fig. 32. It can be seen from the figure that $\dot{q}_{01}''$ decreases with the increase in the depth, d . This is expected, because with the increase in depth, the thermal resistance increases and the decrease in heat transfer occurs. The figure also shows that the rate of decrease in $\dot{q}_{01}''$ is higher at lower values of mass influx. The rate of reduction in $\dot{q}_{01}''$ is smaller for higher values of vapor mass influx. At higher values of mass influx and after a certain value of depth, the heat transfer rate $\dot{q}_{01}''$ attains a constant asymptotic value. These trends are consistent with the previously discussed nature of T_{top} (Fig. 29). To check whether non-dimensionalizing the heat flux $\dot{q}_{01}''$ would give the same trend or not, Fig. 33 is presented. Here $\dot{q}_{01}''$ is normalized by $\dot{Q}_{in}''$. It can be seen from Figs. 32 and 33 that the dimensional and non-dimensional heat flux values have the same qualitative nature.

To show the effect of soil type on the heat flux at the top surface, Fig. 34 is introduced. It can be seen from the figure that for a medium with lower value of thermal conductivity (sand), the heat transfer rate is lower in comparison to the heat transfer rate of a medium with higher thermal conductivity (clay). This is because the thermal resistance is high when the thermal

conductivity is low resulting the lower heat transfer rate. Again, this behavior is consistent with the nature of T_{top} as discussed earlier and shown in Fig. 30. The effect of vapor velocity on the heat transfer rate $\dot{q}_{01}''$ is shown in Fig. 35 for Quincy Sand. As expected, and consistent with the nature of T_{top} (Fig. 31), the heat transfer rate decreases with the decrease of vapor velocity. The extreme case of $V = 0$ is again shown here. The relevance of this case in the present problem is discussed earlier.

The heat transfer rate from dry zone to the moist zone is denoted as $\dot{q}_{12}''$. The variation of $\dot{q}_{12}''$ with the depth of the medium and for different types of soil is shown in Figs. 36 and 37. Here, Fig. 36 is for $V = 5$ mm/s and Fig. 37 is for $V = 50$ mm/s. $\dot{q}_{12}''$ is an important parameter in this analysis. When heat crosses the vapor-liquid interface and if its value is greater than $\dot{q}_{12}'',_{min}$ (defined later), then the liquid water vaporizes and moves upward. To keep the complexity of the problem within the scope of this study, quasisteady state condition is assumed. This allows the imposition of constant vapor velocity as a boundary condition at the interface. $\dot{q}_{12}'',_{min}$ is the minimum amount of heat required to vaporize the liquid water in the moist zone at a rate consistent with the assumed vapor velocity V . It is a function of latent heat of vaporization and soil saturation of the moist zone. It is defined as:

$$\dot{q}_{12}'',_{min} = \rho_l \epsilon s h_{fg} \left[\frac{V(\rho_v/\rho_l)}{s - (\rho_v/\rho_l)} \right] \quad (54)$$

where,

- ρ_l = density of liquid water at saturation
- ρ_v = density of water vapor at saturation
- V = vapor velocity
- h_{fg} = latent heat of vaporization
- s = soil saturation
- ϵ = porosity of the soil

It can be seen from Figs. 36 and 37 that the heat transfer rate from the dry zone to the moist zone decreases with the increase of the depth of the medium. This is quite expected, because with the increased depth, the thermal resistance increases, resulting in a decrease of $\dot{q}_{12}''$. These figures also show that clay has a higher heat transfer rate in comparison to sand. Again, this is because the thermal conductivity of clay is higher than that of sand. There seems to be a significant disagreement between the soil science literature [Campbell *et al.* (1994)] and common engineering references [Incropera and DeWitt (1990)] regarding the relative thermal conductivity values of sand and clay! We have here used engineering references, but the figures are not undisputed. Fig. 38 shows the effect of vapor velocity V on the values of $\dot{q}_{12}''$ for different depths of the medium. Quincy sand with a radiant heating rate of $\dot{Q}_{in}'' = 70$ kW/m² is used in this figure. It can be seen that $\dot{q}_{12}''$ decreases with the increase of vapor velocity, V . This is because the vapor flow opposes the direction of heat flow. So, at the interface, with the

increase in vapor velocity, less heat can be transmitted. This observation is very much consistent with the earlier discussion about heat flux vectors shown in Figs. 26(c) and 28(b). From the profiles of Fig. 38 it can also be seen that the effect of vapor velocity on $\dot{q}_{12}''$ is more profound at lower values of the medium depth, d .

The horizontal line on Figs. 36 and 37 and marked as "cut off line" is the value of $\dot{q}_{12}'',_{\min}$. This value is calculated by using equation (3) and assuming a 10% saturation of moist zone soil. Recall that $\dot{q}_{12}'',_{\min}$ is the minimum amount of heat required to maintain vapor velocity V . That means, portions of the curves of these two figures below the "cut off line" are not relevant. Below this line, the values of $\dot{q}_{12}''$ are too small to produce the assumed flow rate of water vapor.

The ratios of the heat fluxes ($\dot{q}_{01}''/\dot{q}_{12}''$) for different values of radiation heat source intensity ($\dot{Q}_{in}''$) and vapor velocity (V) are shown in Figs. 39 and 40. Here, Fig. 39 is for a dry zone depth, $d = 0.5$ cm and Fig. 40 is for $d = 1.5$ cm. It can be seen from these figures that with the increase in the value of $\dot{Q}_{in}''$, the heat flux ratio ($\dot{q}_{01}''/\dot{q}_{12}''$) remains constant at lower values of vapor velocity. At higher values of V , the ratio decreases slightly. By comparing the numerical values of the heat flux ratio (for the same value of V) in Figs. 39 and 40, it can be seen that the ratio increases with the increase in depth. This is primarily because of more decrease in $\dot{q}_{12}''$ as compared to the decrease in $\dot{q}_{01}''$. It can also be seen from these figures that at any particular value of $\dot{Q}_{in}''$ and d , the heat flux ratio increase with the increase in V . This is expected because $\dot{q}_{01}''$ increases and $\dot{q}_{12}''$ decreases with increasing V . From the earlier figures related to T_{top} , $\dot{q}_{01}''$ and $\dot{q}_{12}''$ (Figs. 29, 32, 36, and 37) it can be seen that values of these parameters increase with the increase in $\dot{Q}_{in}''$.

D. Summary of the Results and Conclusion

It is observed that by keeping the radiant heat source intensity constant, the temperature at the top surface increases as the thickness of the dry zone increases. The top surface temperature decreases with the increase of vapor influx from the bottom. This vapor flux opposes the overall direction of heat transfer through the heated space. For higher values of vapor influx, the top surface temperature reaches an asymptotic value with increasing thickness of the medium. The amount of heat that is being transmitted from the radiant heater to the soil at the top surface also decreases with the growing thickness of the medium. At higher values of vapor influx and after a certain value of the thickness of the dry zone, the amount of heat flux at the top surface reaches a constant asymptotic value.

When the incident power flux of the radiant heater at the top is kept constant, the amount of heat that is being transmitted through the dry-moist interface decreases with the increasing thickness of the medium. The heat transfer through this interface also declines with the increasing flow of the vapor through the dry zone. On the other hand, the amount of vapor generation and subsequent movement of the interface line are dependent on the amount of heat that is being transmitted from the dry zone to the moist zone. These two opposing phenomena have a negative effect on the overall mechanics of the heat transfer. Therefore, an accurate prediction

of heat transfer through the dry zone is crucial. In particular, this is very important to predict the temperature distribution of the moist zone.

A feature of the process of heat transfer in moist soil that is improperly modeled here is the transition between dry and moist soil. The assumption that the moisture content of the moist soil is invariant except for evaporation at a fixed temperature is violated in laboratory experiments with simulated fire heating when the dry soil zone is only a few centimeters deep [Campbell *et al* 1995]. The interface between the two zones becomes an extended transition region of varying moisture content because of rapid evaporation at and above about 80 C and probably because of liquid phase moisture migration downward in response to the temperature gradient. So, it can be concluded that a model for the processes of heat and moisture transport in fire heated moist soils that is expected to make accurate predictions of temperature fields extending significantly below the nominal boiling point of water must have at least the following two attributes: (1) It must model heat transfer through the upper, dry soil zone with high fidelity, and (2) it must include an accurate representation of the response of the liquid phase moisture to temperature gradients at high (but below boiling) temperature.

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Fig. 26. Heat flux vectors for a medium at different times.

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Fig. 36. Variation of heat flux from the dry zone to moist zone ($\dot{q}_{12}''$) with depth for different types of soils (sand and clay); $V = 0.005 \text{ m/s}$.

Fig. 37. Variation of heat flux from the dry zone to moist zone ($\dot{q}_{12}''$) with depth for different types of soils (sand and clay); $V = 0.05 \text{ m/s}$.

Fig. 38. The effect of vapor velocity (V) on the heat flux from the dry zone to moist zone ($\dot{q}_{12}''$); Quincy Sand, $\dot{Q}_{in}'' = 70 \text{ kW/m}^2$.

Fig. 39. Heat flux ratio ($\dot{q}_{01}''/\dot{q}_{12}''$) for a medium (sand) with dry zone depth, $d = 0.5 \text{ cm}$.

Fig. 40. Heat flux ratio ($\dot{q}_{01}''/\dot{q}_{12}''$) for a medium (sand) with dry zone depth, $d = 1.5 \text{ cm}$.

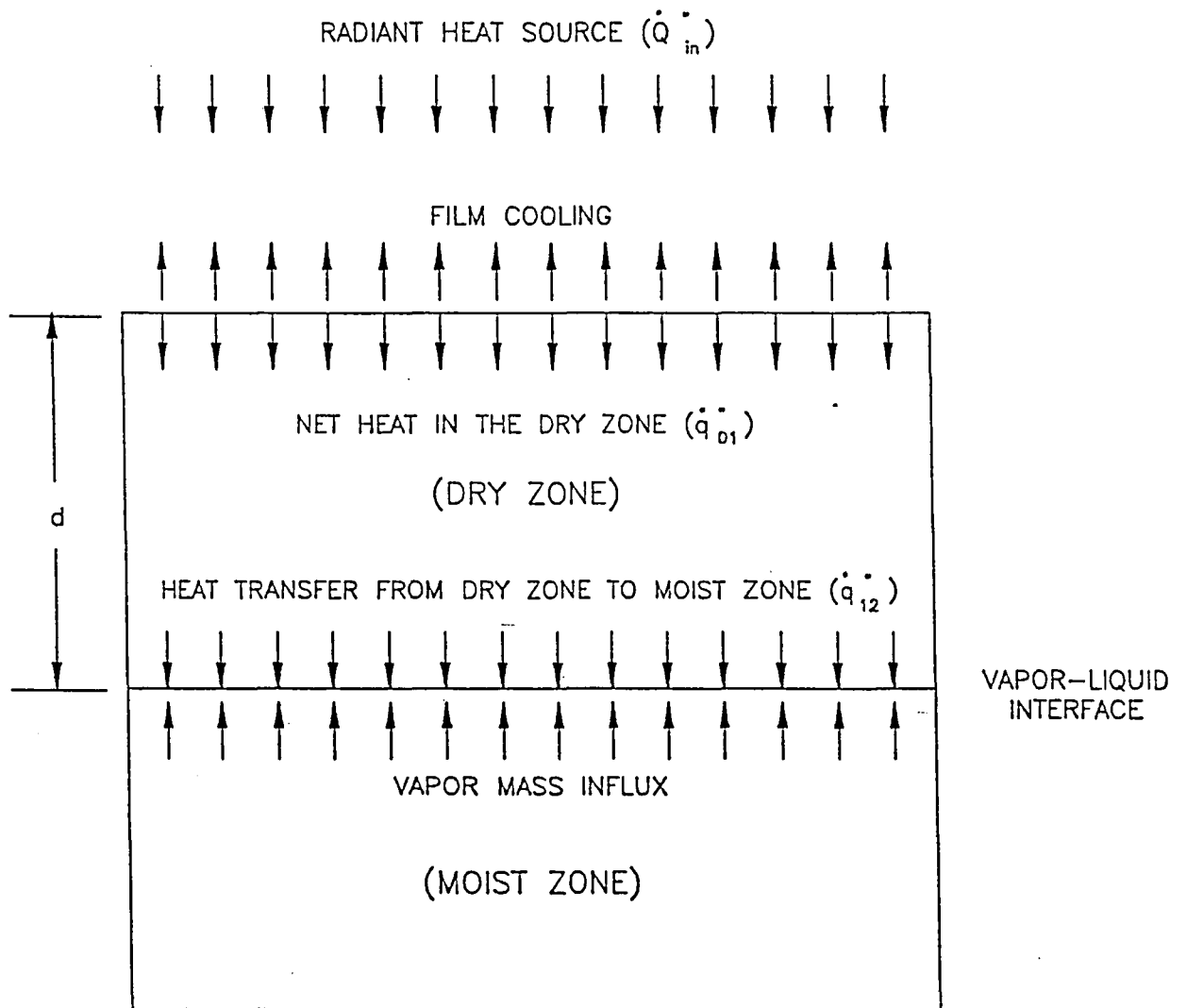


Fig. 1. Schematic diagram of the heated zones

TIME REQUIRED FOR SUDDENLY HEATED SURFACE TO REACH BOILING

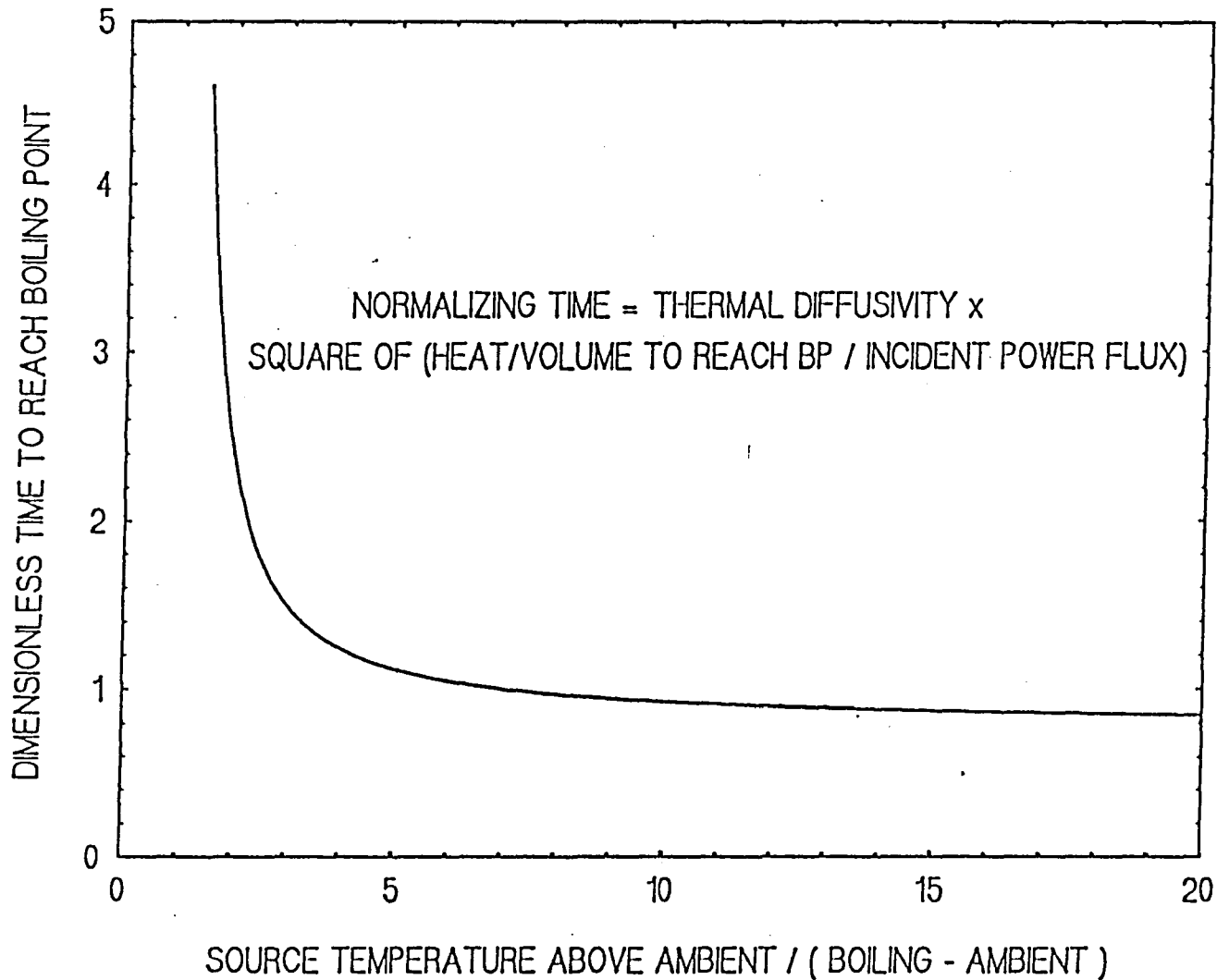


Fig. 2. Time required for the surface of a halfspace to reach a specified ("boiling") temperature after it is suddenly exposed to heating by a radiant source of specified temperature and cools by film heat transfer at a rate proportional to the temperature difference between the heated surface and the ambient (initial) temperature.

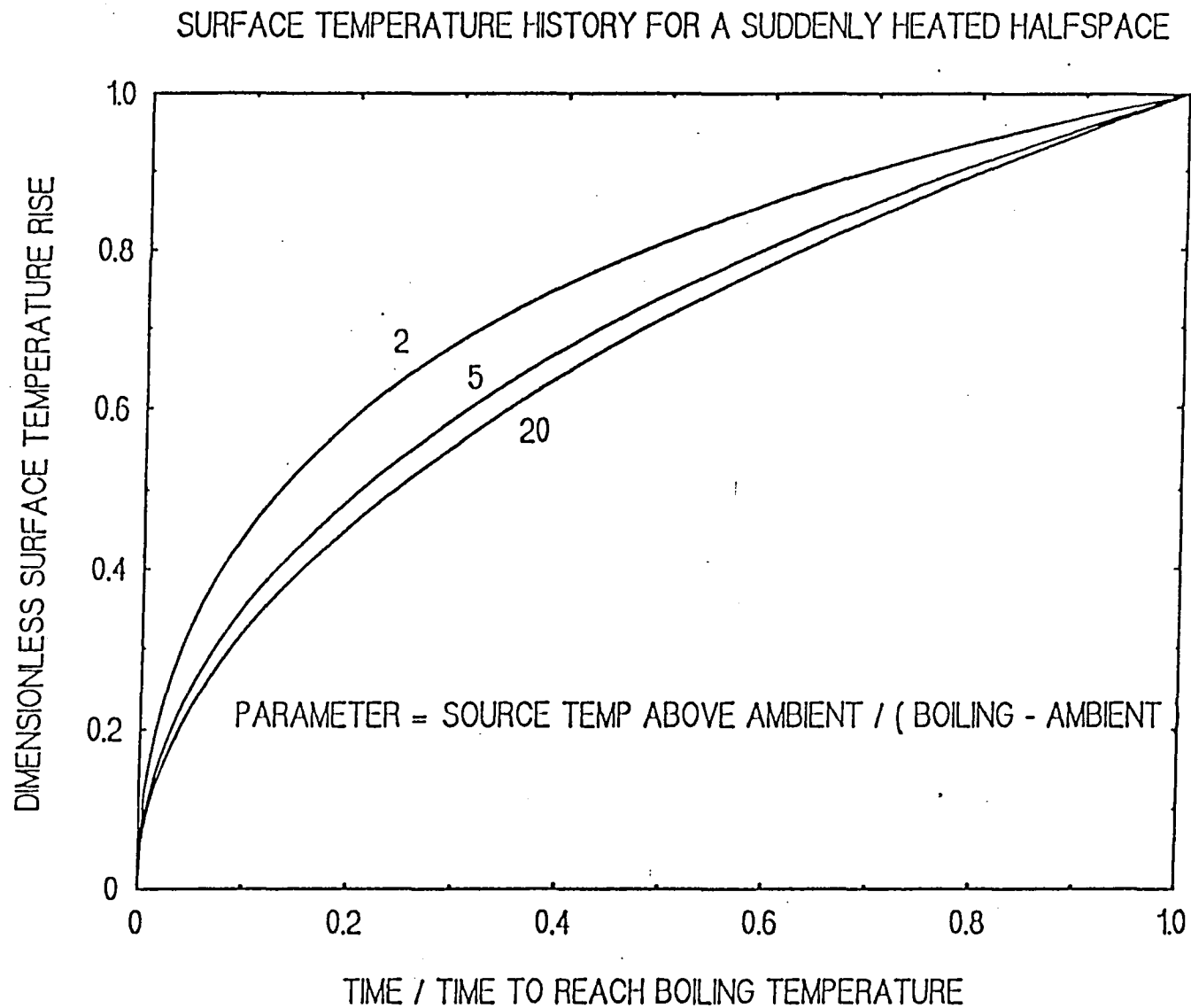


Fig. 3. Temperature histories of the surfaces of halfspaces heated by radiant sources of different temperatures.

A line graph showing the dimensionless temperature rise as a function of the distance from the surface normalized by the diffusion length. The y-axis is labeled 'DIMENSIONLESS TEMPERATURE RISE' and ranges from 0 to 1.0. The x-axis is labeled 'DISTANCE FROM SURFACE / DIFFUSION LENGTH' and ranges from 0 to 2.0. Two curves are plotted, both starting at (0, 1.0) and decaying towards zero. The upper curve is labeled '2' and the lower curve is labeled '1'. Text within the graph area states: 'PROFILE AT THE INSTANT SURFACE REACHES BOILING' and provides the formula: $12 = (\text{SOURCE} - \text{AMBIENT}) / (\text{BOILING} - \text{AMBIENT})$.

Distance from Surface / Diffusion Length	Dimensionless Temperature Rise (Curve 1)	Dimensionless Temperature Rise (Curve 2)
0.0	1.00	1.00
0.5	0.40	0.45
1.0	0.10	0.12
1.5	0.02	0.03
2.0	0.00	0.01

Fig. 4. Profile of temperature rise with depth into a halfspace with a suddenly heated surface.

RELAXATION OF INITIAL TEMPERATURE DISTRIBUTION

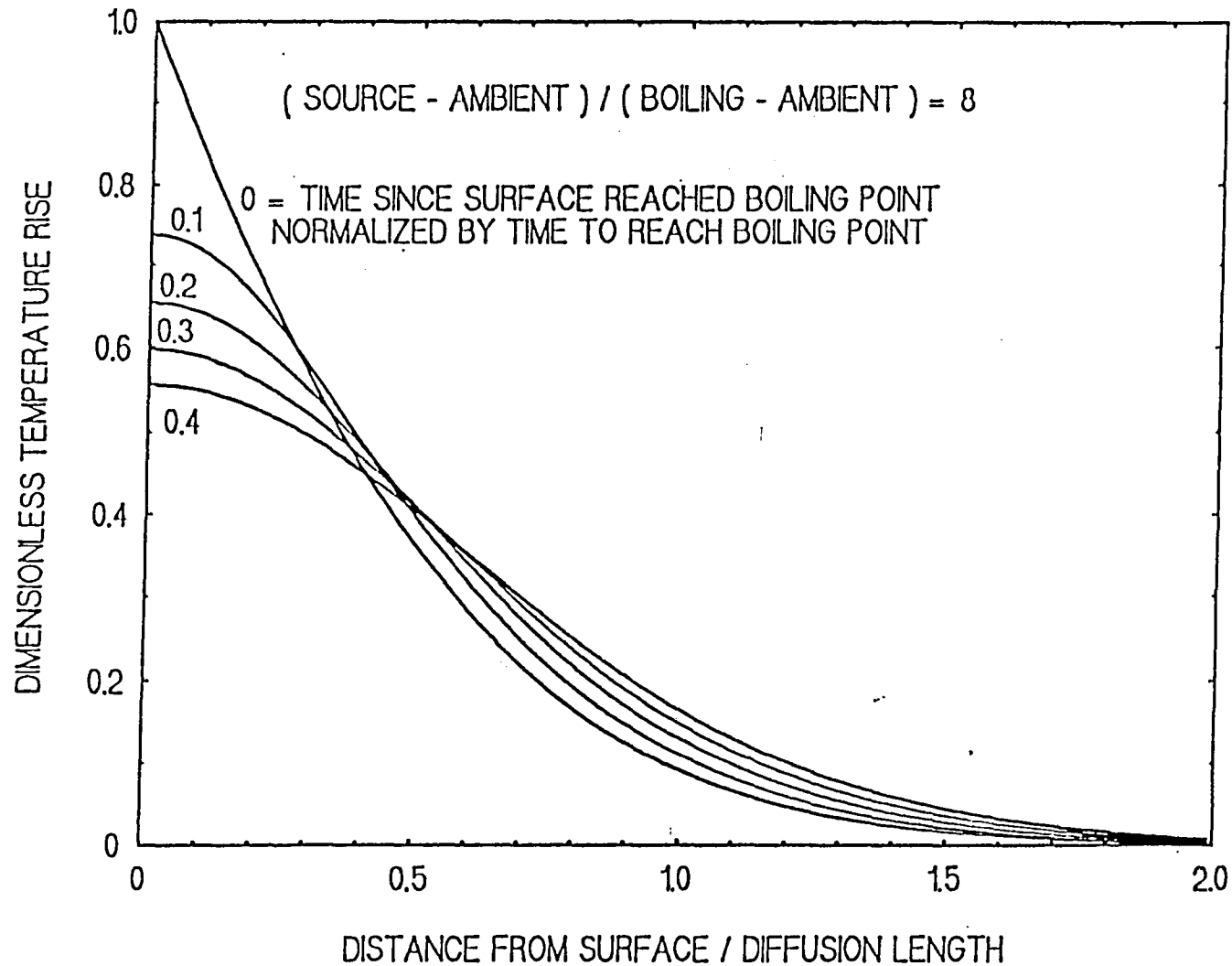


Fig. 5. Diffusion of a temperature field like that in figure 4 if the heating suddenly stops and no heat in the halfspace is allowed to escape through the surface.

RELAXATION OF INITIAL TEMPERATURE DISTRIBUTION

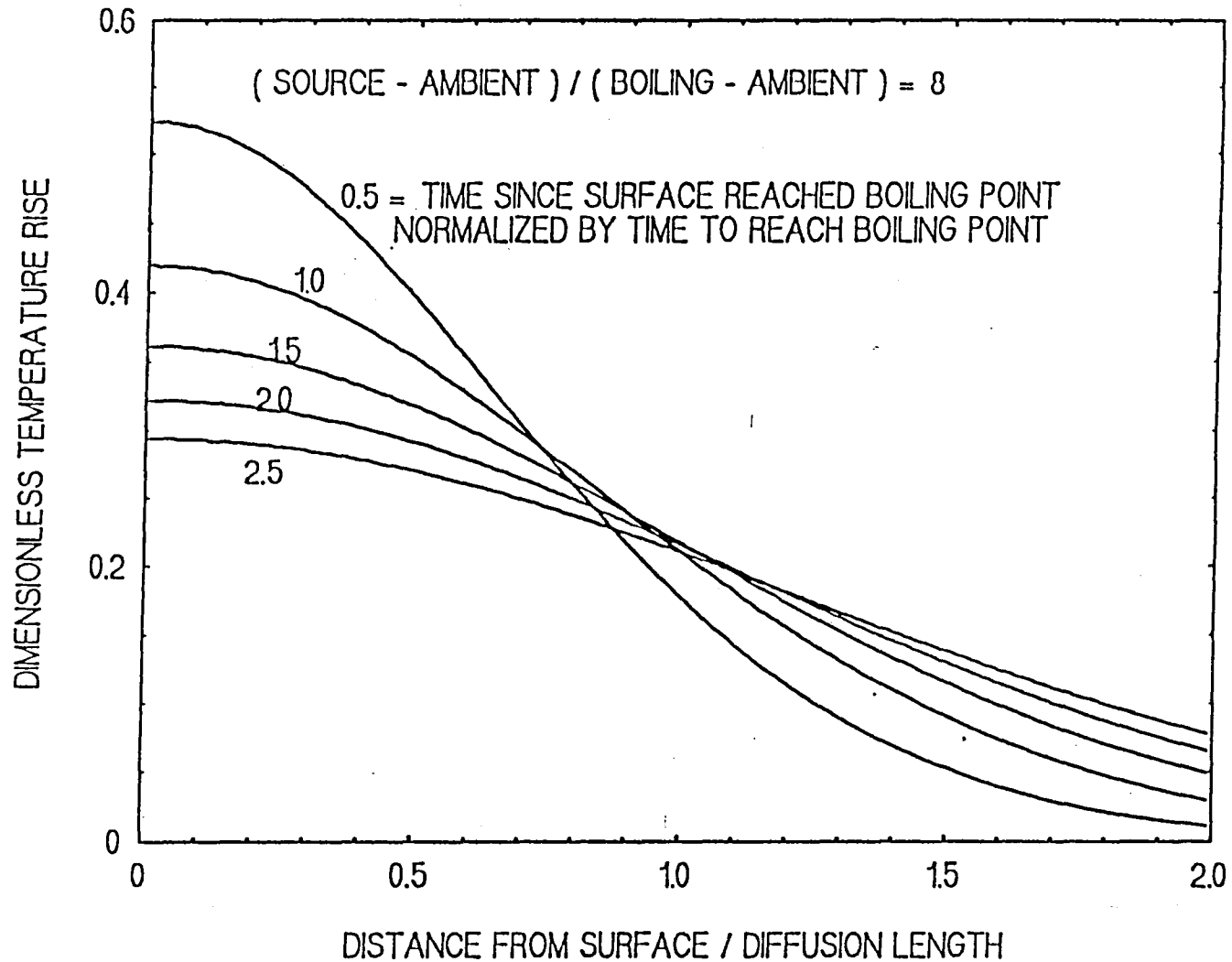


Fig. 6. Extension of figure 5 to longer time.

TEMPERATURE HISTORY OF HEATED SURFACE

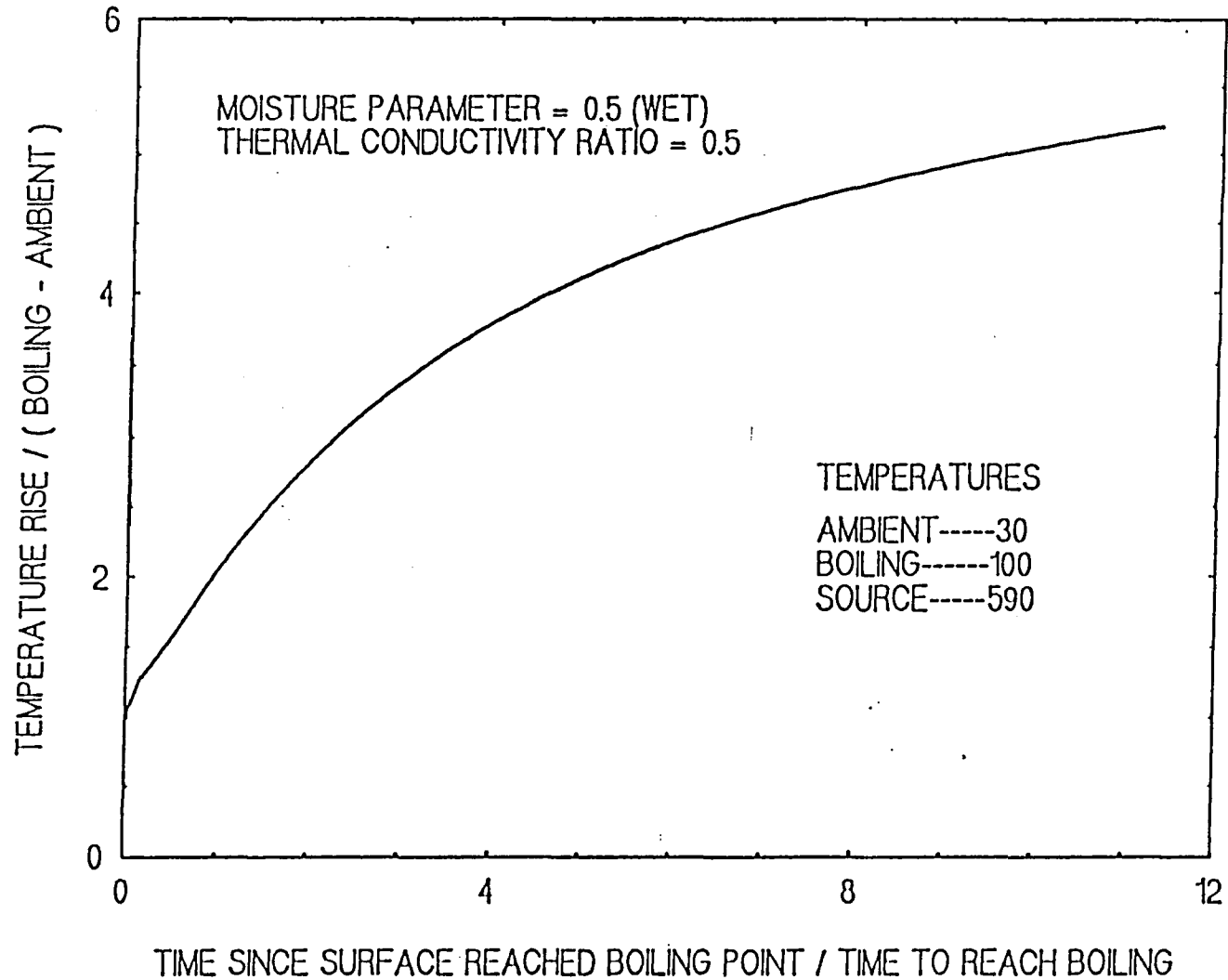


Fig. 7. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to a simple model for the growth of the dry soil zone. The soil in this case is fully saturated sand.

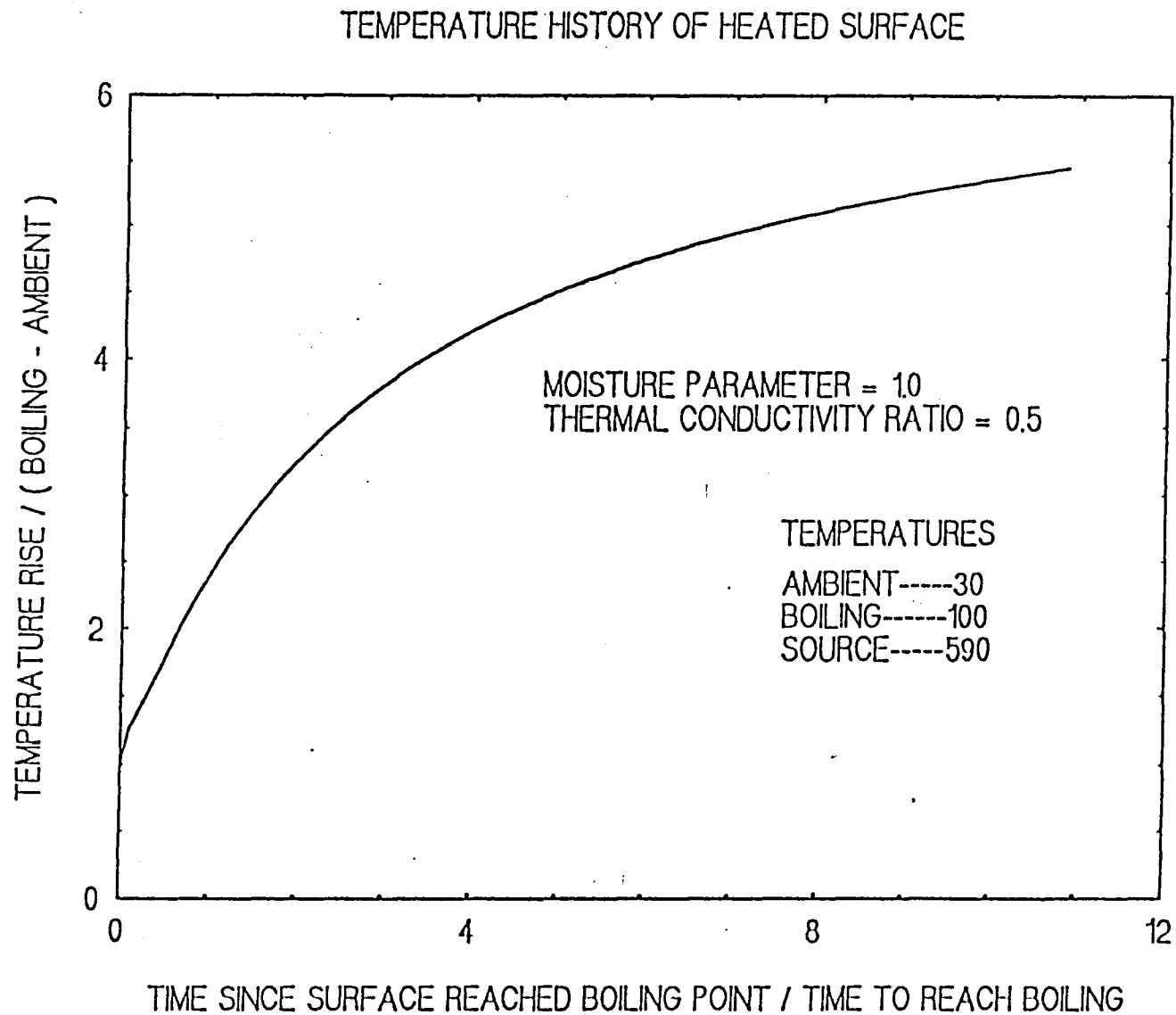


Fig. 8. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to a simple model for the growth of the dry soil zone. The soil in this case is 50% saturated sand.

TEMPERATURE HISTORY OF HEATED SURFACE

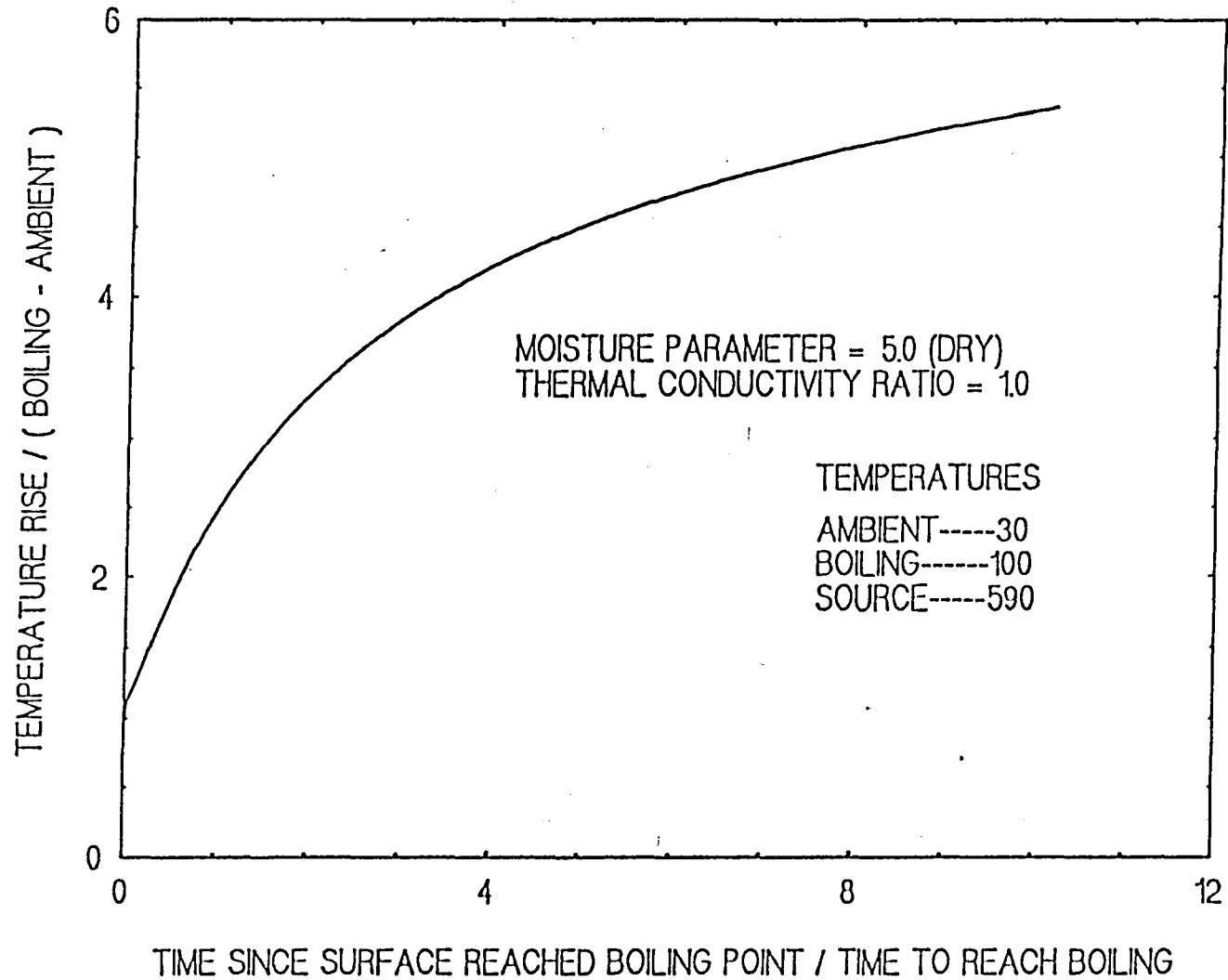


Fig. 9. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to a simple model for the growth of the dry soil zone. The soil in this case is very dry sand.

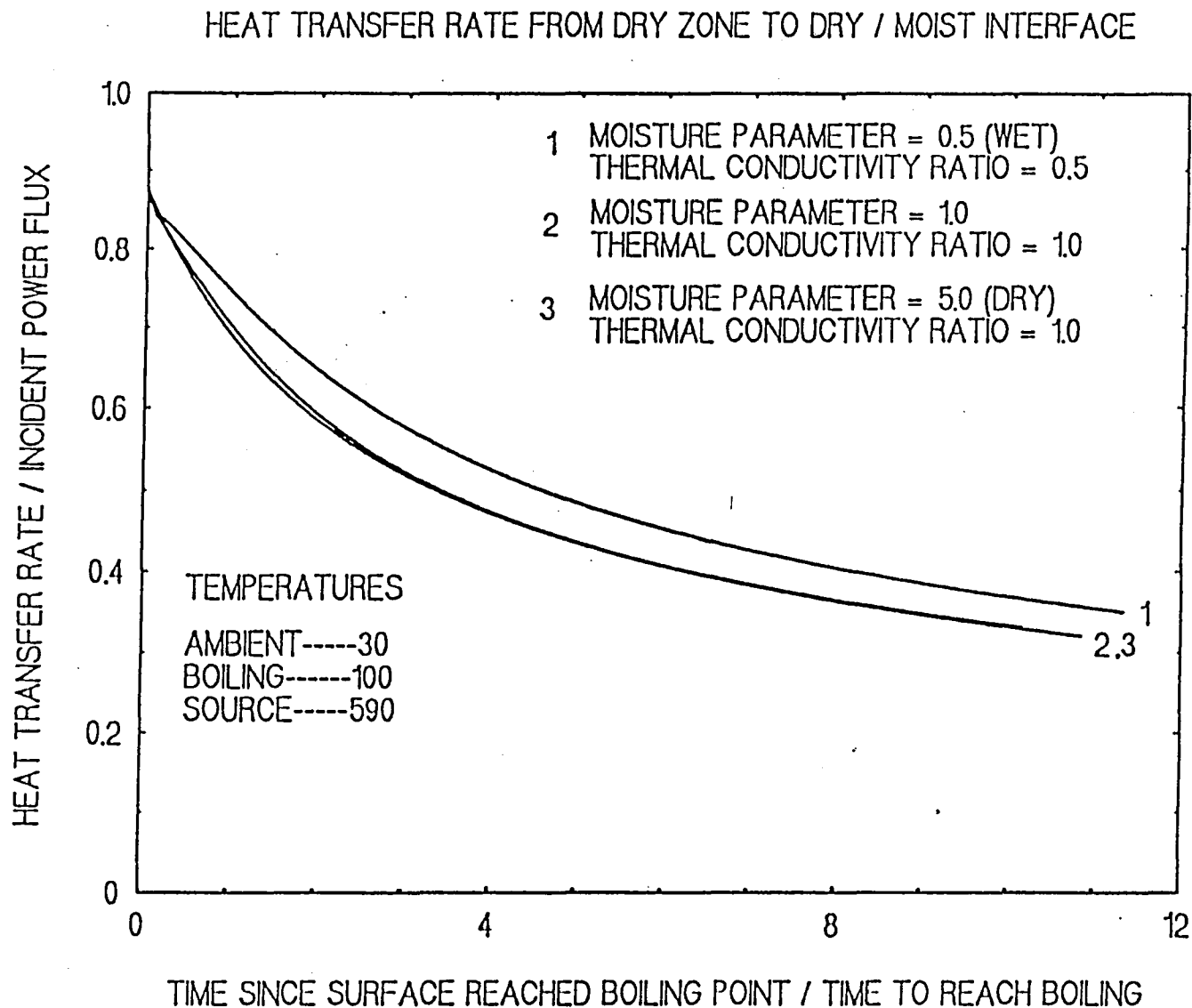


Fig. 10. Fraction of the power flux onto the dry soil zone surface that is transferred to the interface separating the dry and moist soil zones. Three soil moisture conditions are shown. The heat source is the same in each case.

LOCATION OF DRY / MOIST INTERFACE

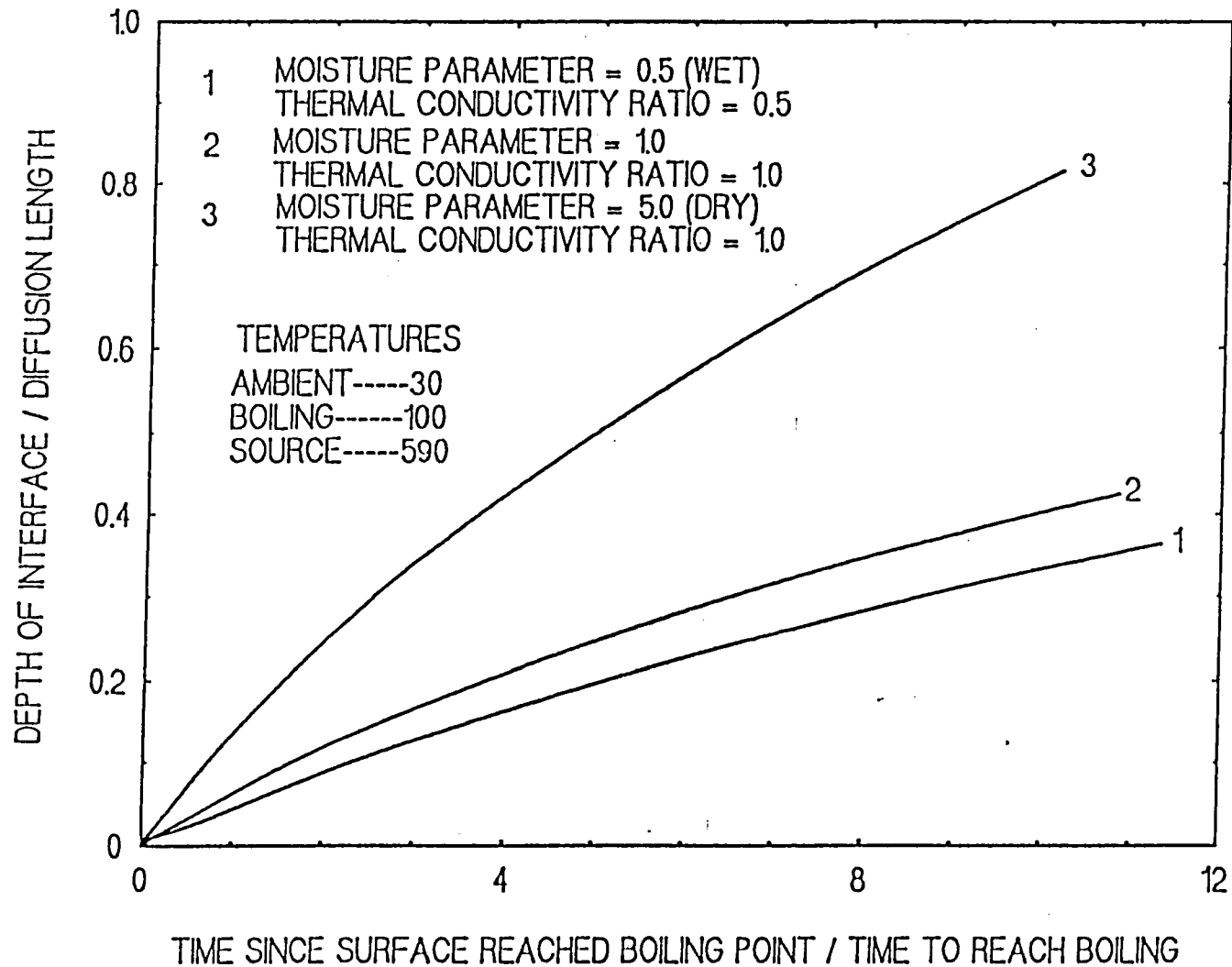


Fig. 11. Predicted depth of the interface between dry and moist soil zones as a function of time for three soil moisture conditions subject to the same heat source.

VELOCITY OF DRY / MOIST INTERFACE

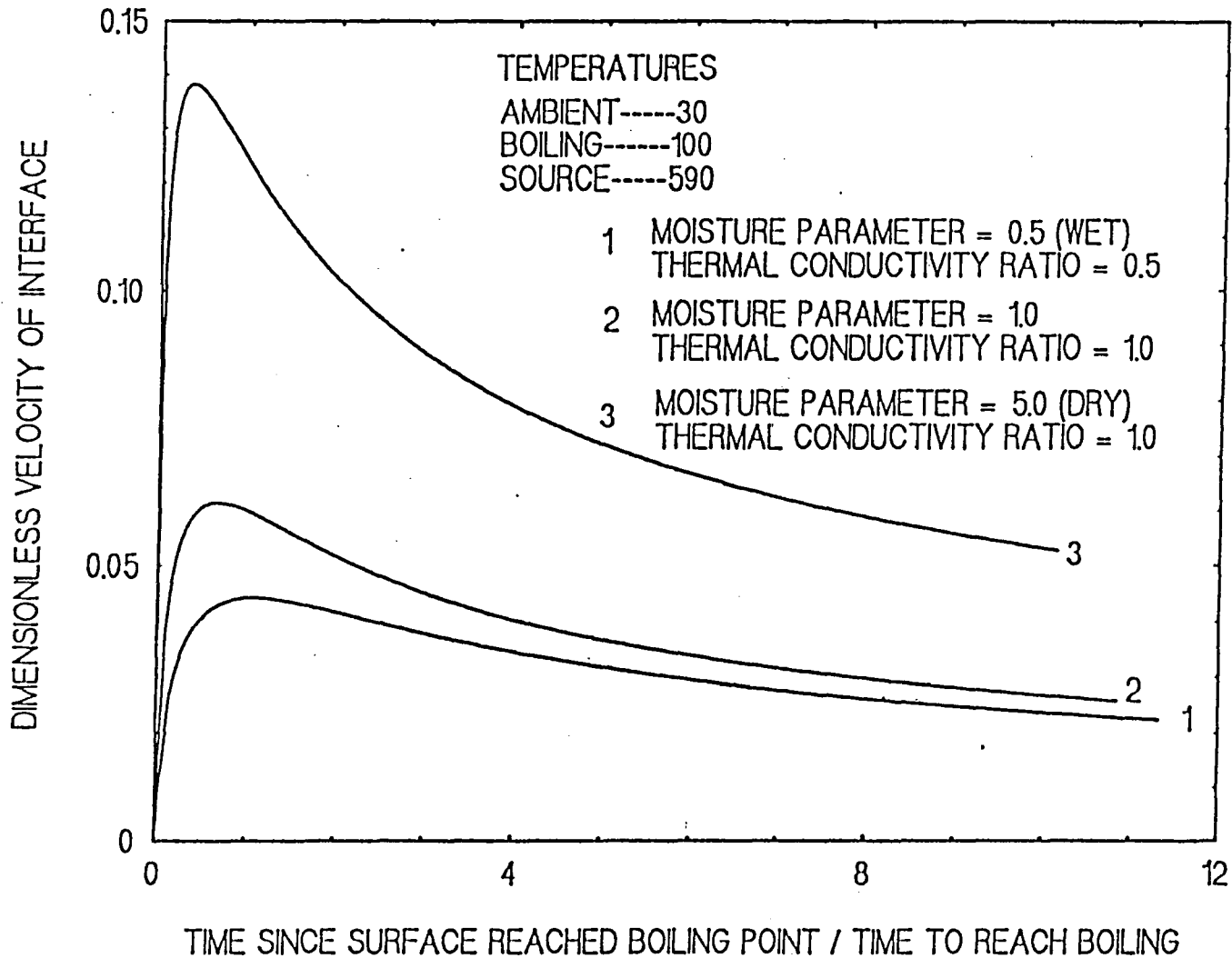


Fig. 12. Predicted velocity of the interface between dry and moist soil zones as a function of time for three soil moisture conditions subject to the same heat source.

QUASISTEADY APPROXIMATION TEMPERATURE PROFILES

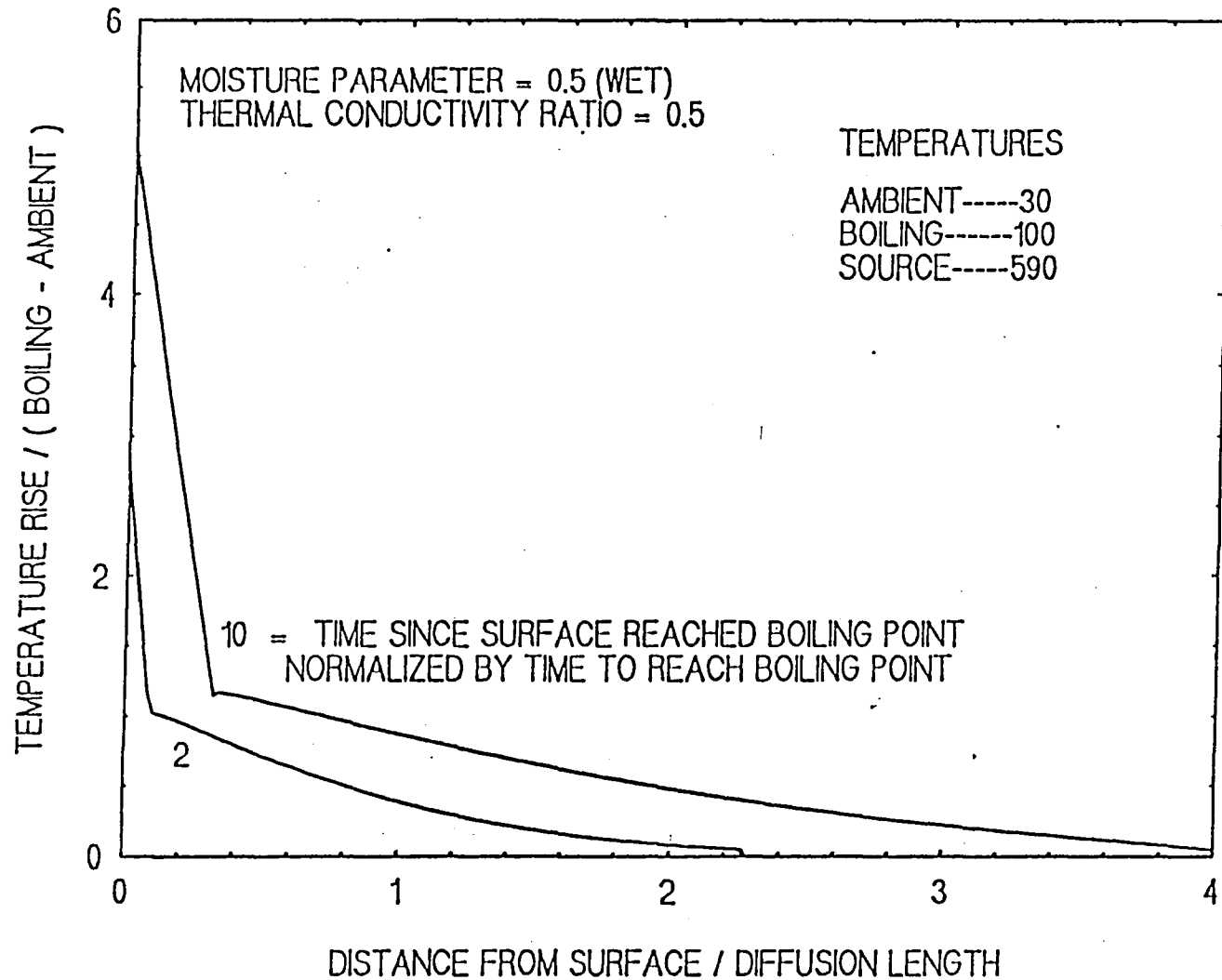


Fig. 13. Snapshot profiles of temperature rise as a function of distance into the soil. The soil in this case is fully saturated sand.

QUASISTEADY APPROXIMATION TEMPERATURE PROFILES

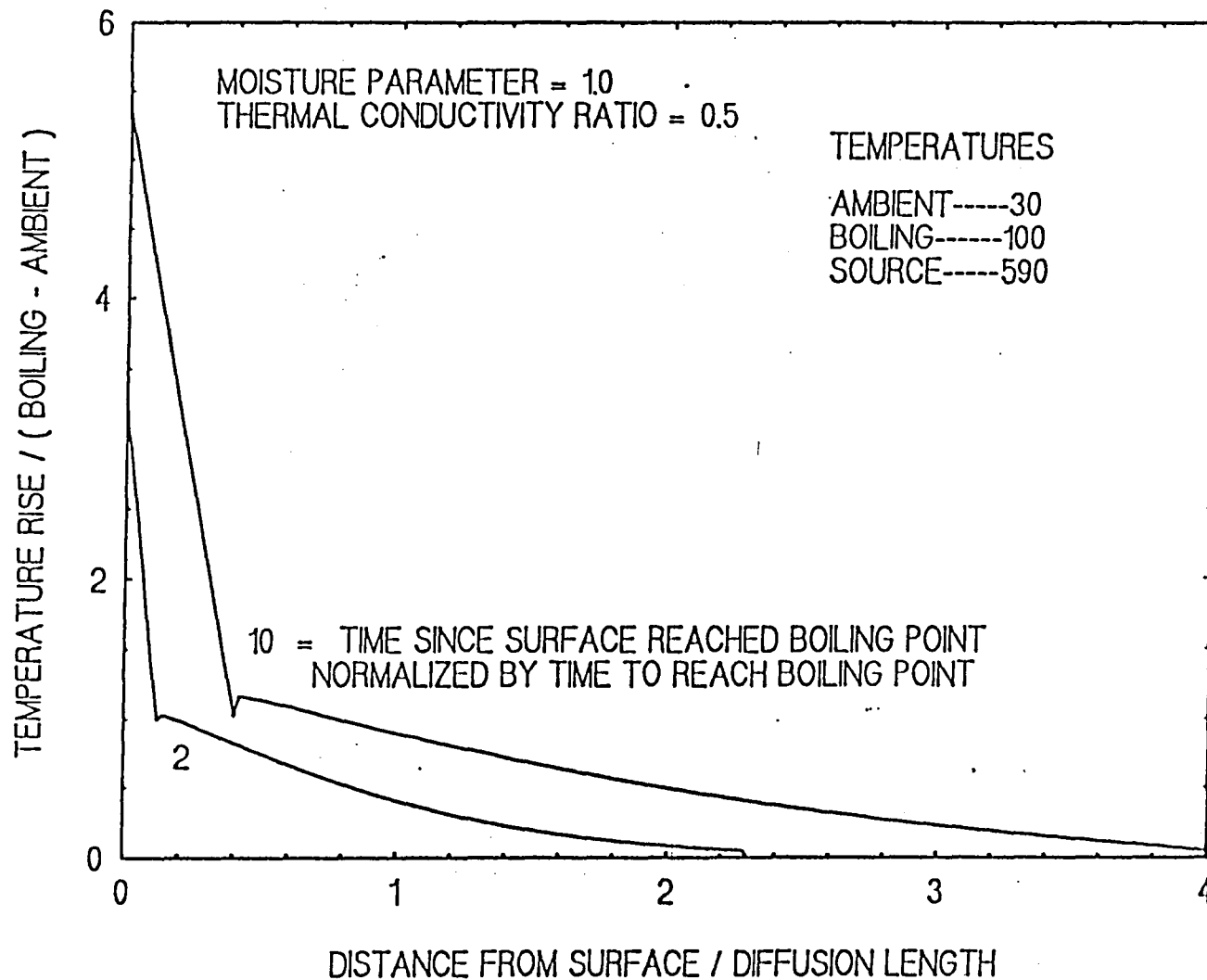


Fig. 14. Snapshot profiles of temperature rise as a function of distance into the soil. The soil in this case is 50% saturated sand.

QUASISTEADY APPROXIMATION TEMPERATURE PROFILES

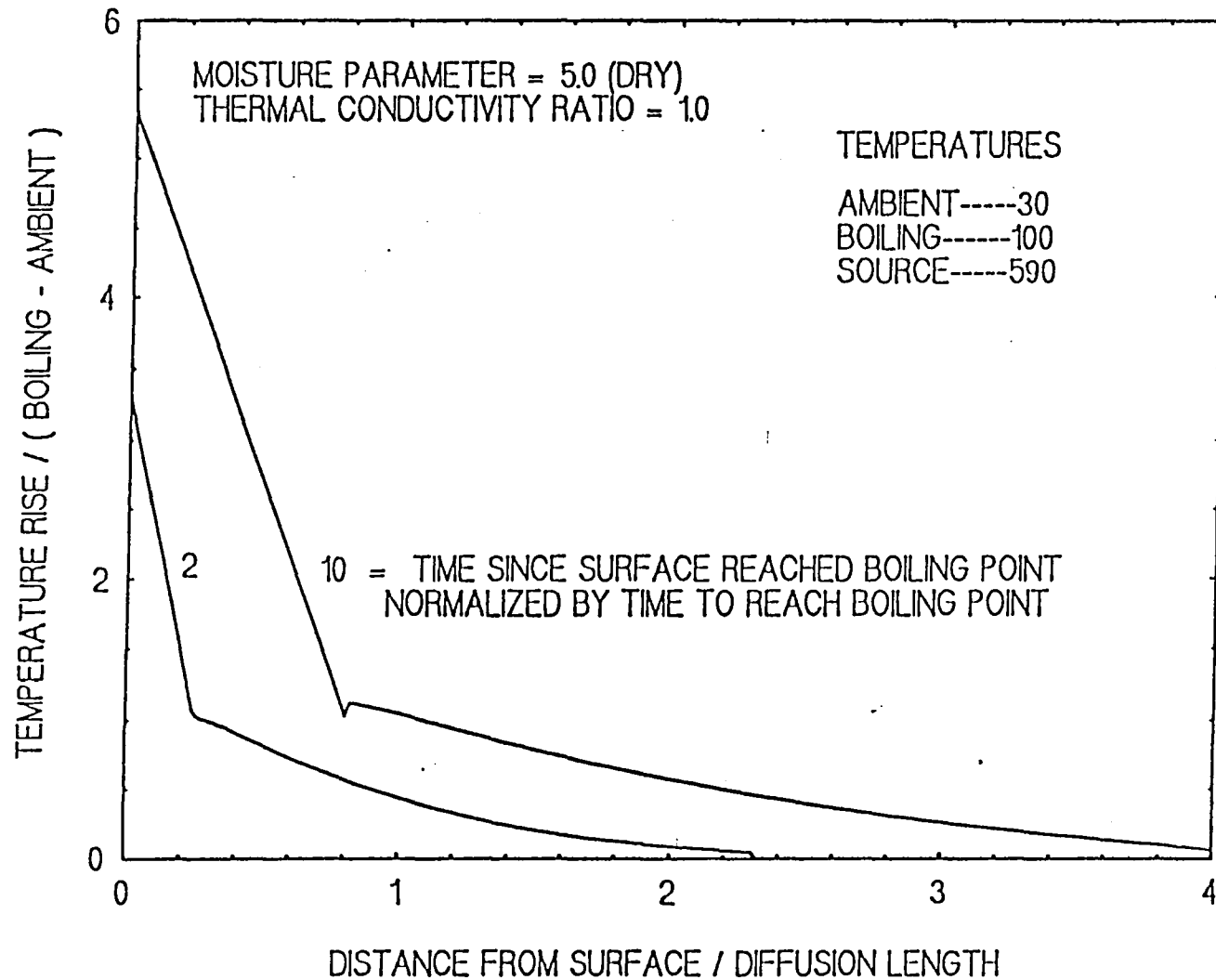


Fig. 15. Snapshot profiles of temperature rise as a function of distance into the soil. The soil in this case is very dry sand.

ASYMPTOTIC TEMPERATURE HISTORY OF HEATED SURFACE

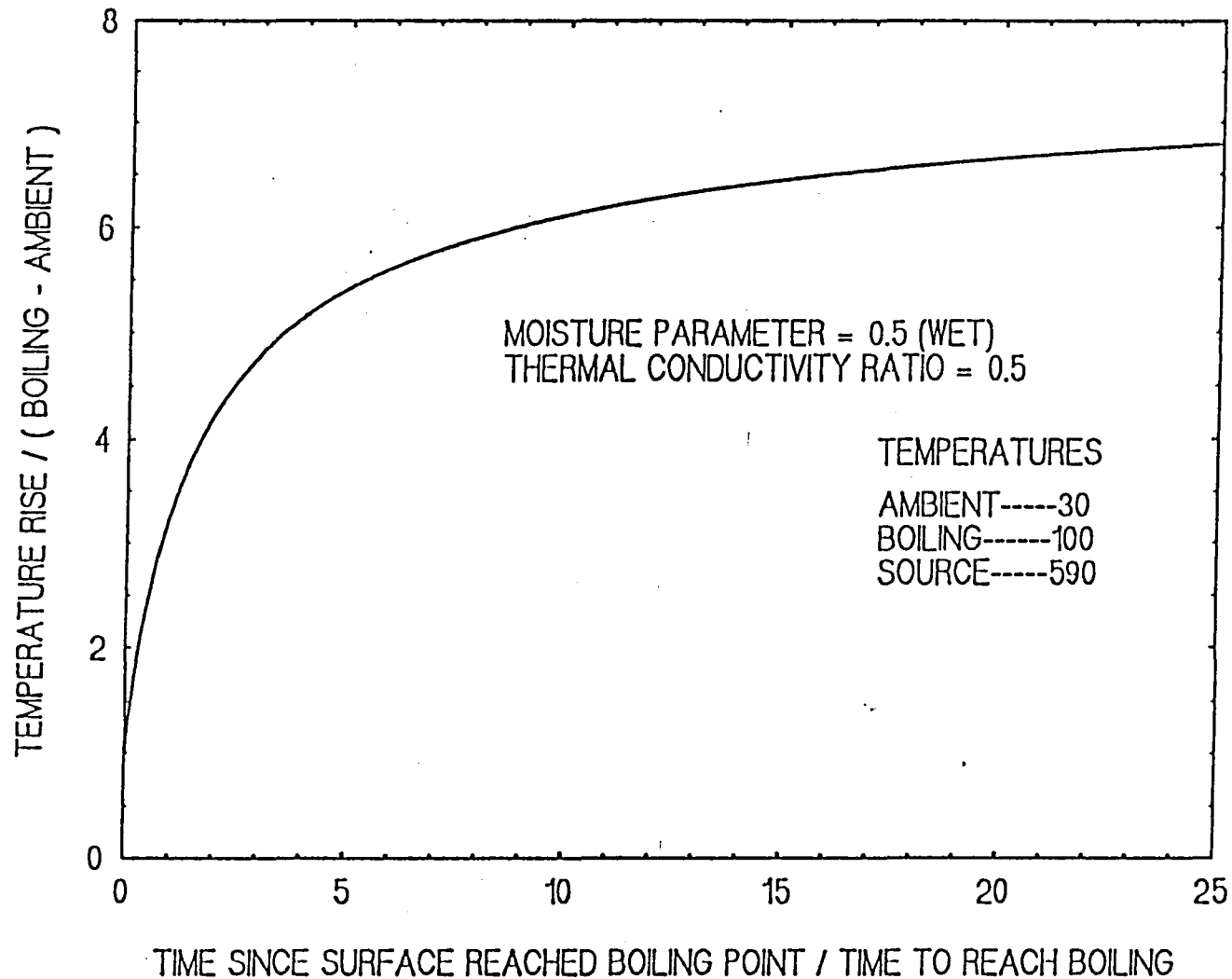


Fig. 16. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to an asymptotic model for the growth of the dry soil zone (compare to figure 7). The soil in this case is fully saturated sand.

ASYMPTOTIC TEMPERATURE HISTORY OF HEATED SURFACE

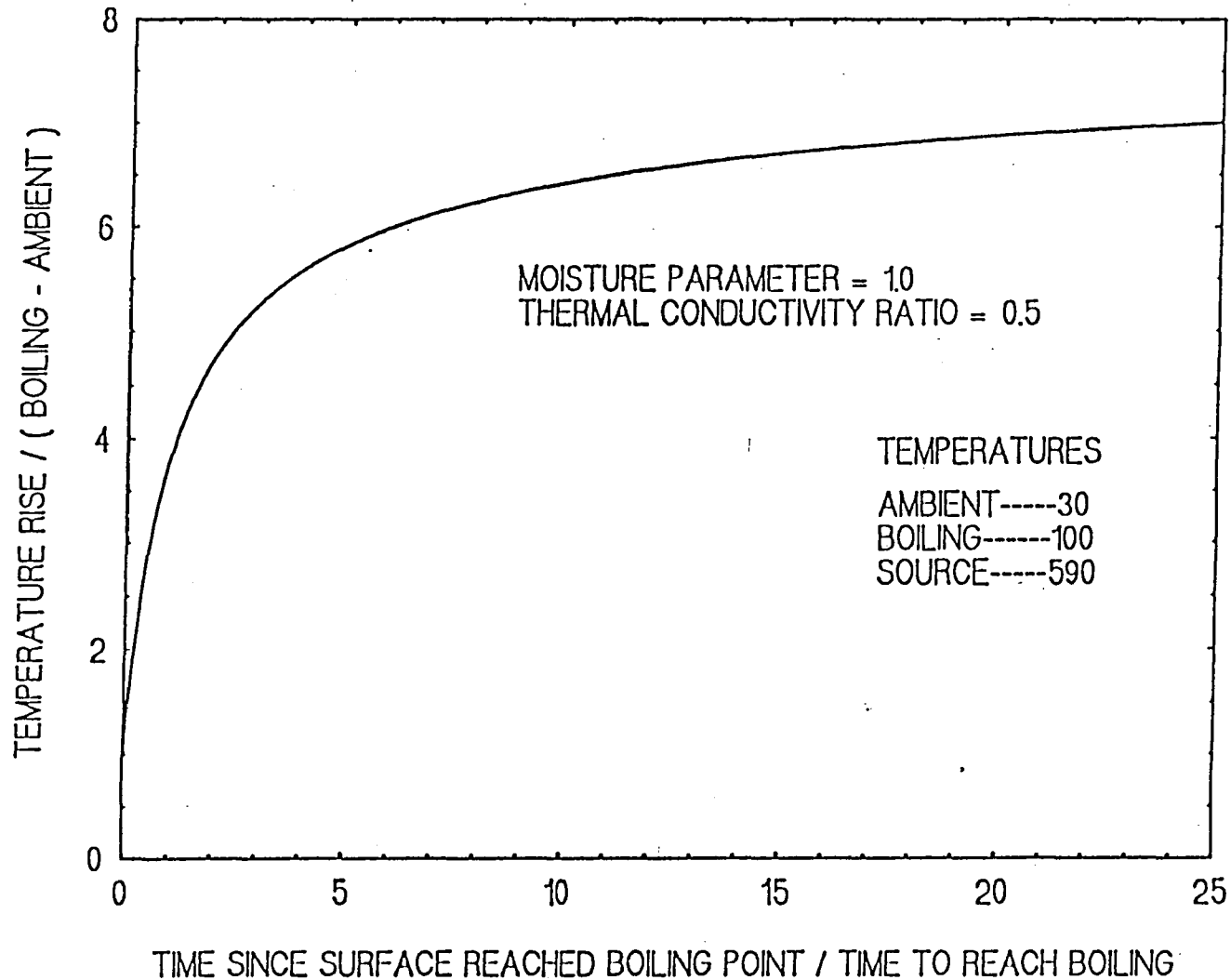


Fig. 17. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to an asymptotic model for the growth of the dry soil zone (compare to figure 8). The soil in this case is 50% saturated sand.

ASYMPTOTIC TEMPERATURE HISTORY OF HEATED SURFACE

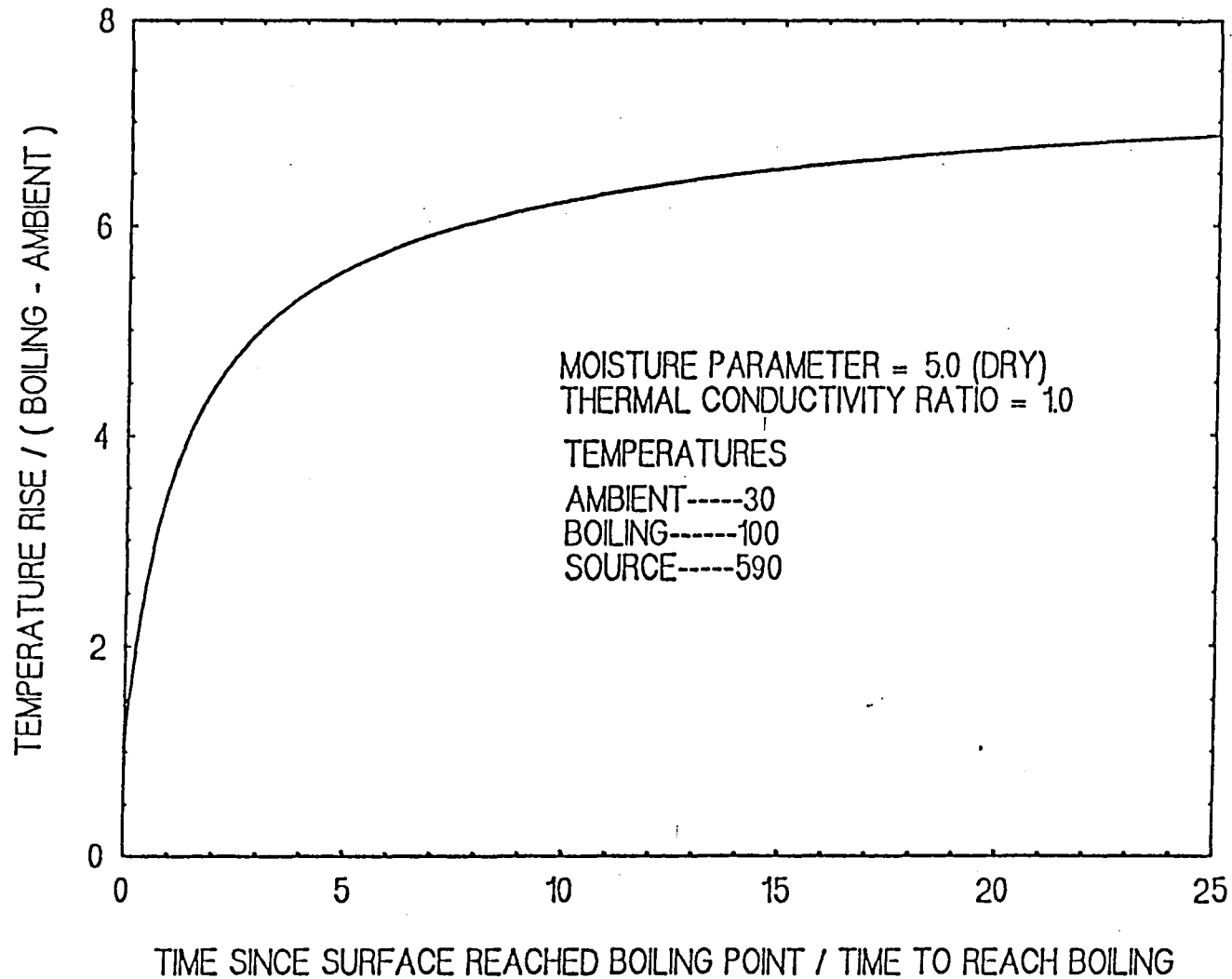


Fig. 18. Temperature history of the radiantly heated surface of a halfspace beyond the time at which the surface reaches the boiling point, according to an asymptotic model for the growth of the dry soil zone (compare to figure 9). The soil in this case is very dry sand.

ASYMPTOTIC HEAT TRANSFER RATE FROM DRY ZONE TO DRY / MOIST INTERFACE

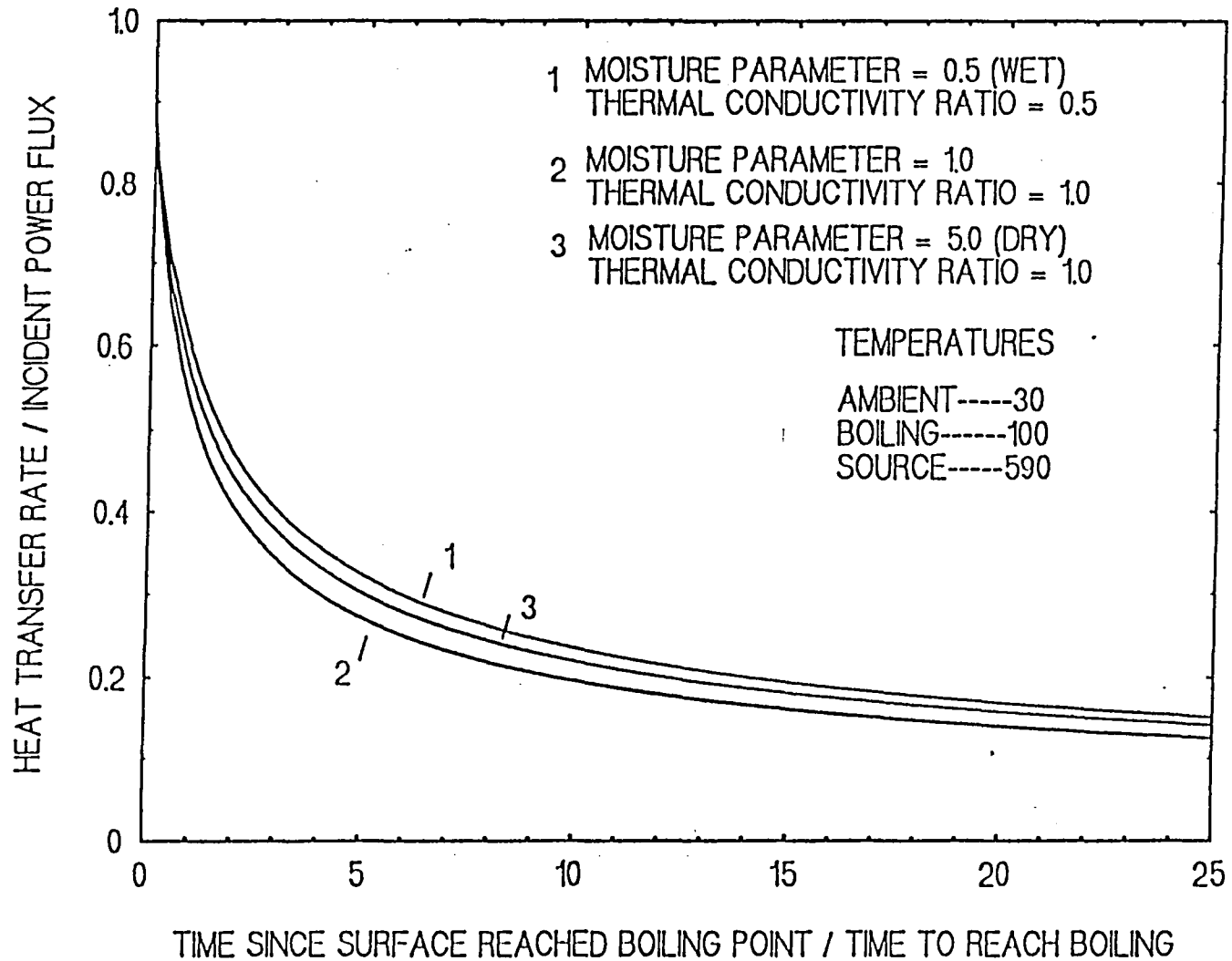


Fig. 19. Fraction of the power flux onto the dry soil zone surface that is transferred to the interface separating the dry and moist soil zones, using an asymptotic model for the growth of the dry soil zone (compare to figure 10). Three soil moisture conditions are shown. The heat source is the same in each case.

ASYMPTOTIC LOCATION OF DRY / MOIST INTERFACE

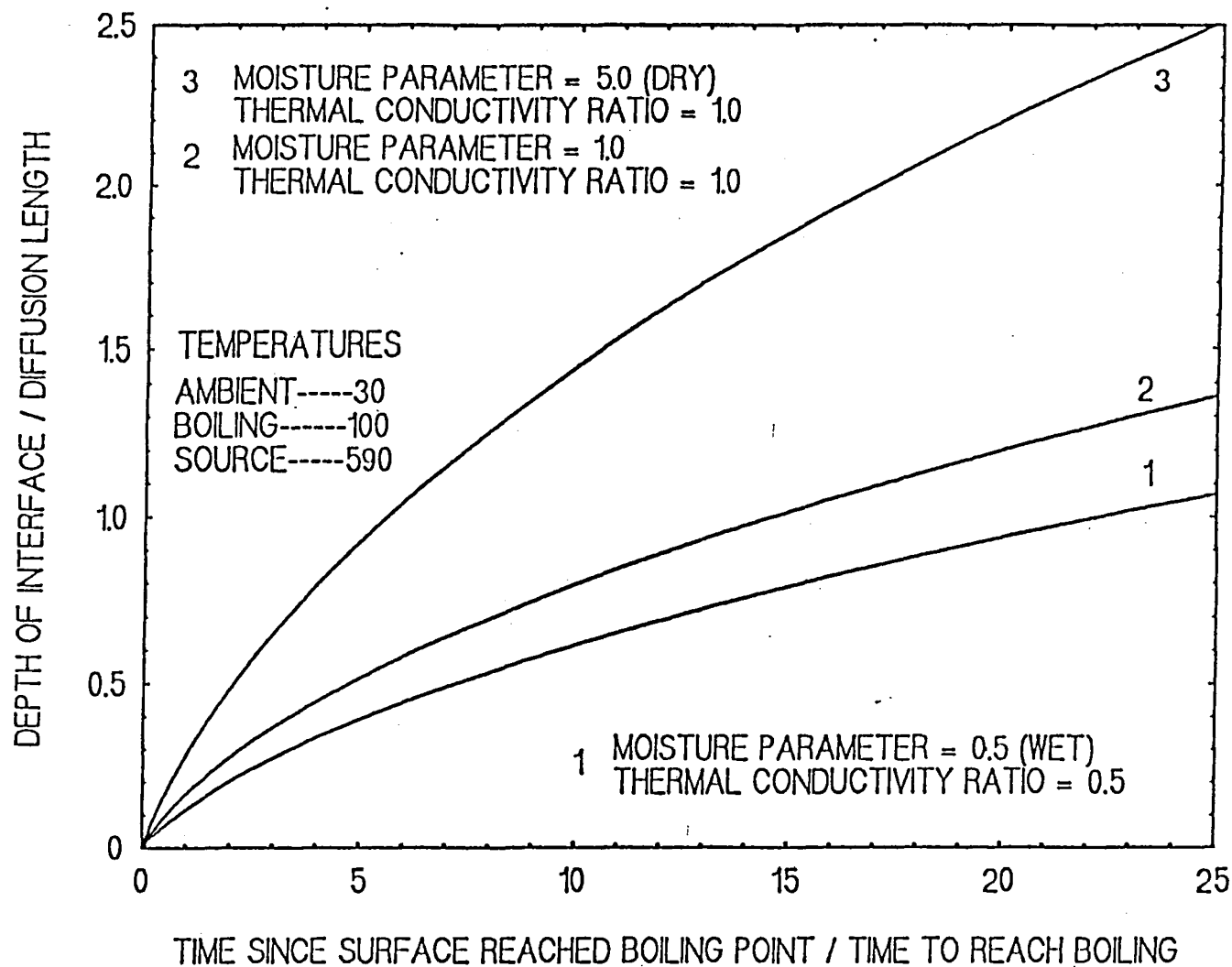


Fig. 20. Predicted depth of the interface between dry and moist soil zones as a function of time for three soil moisture conditions subject to the same heat source, using an asymptotic model for the growth of the dry soil zone (compare to figure 11).

ASYMPTOTIC VELOCITY OF DRY / MOIST INTERFACE

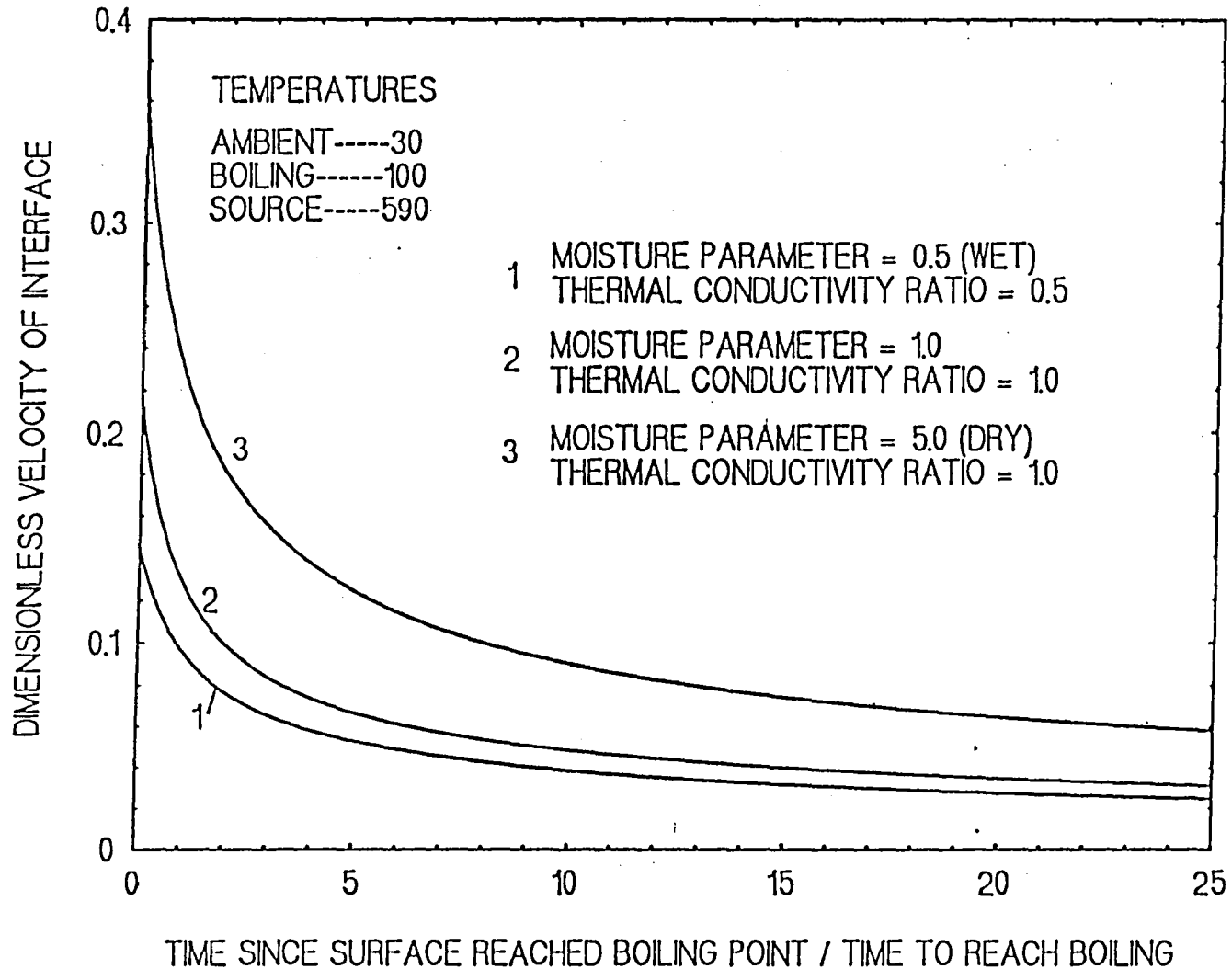


Fig. 21. Predicted velocity of the interface between dry and moist soil zones as a function of time for three soil moisture conditions subject to the same heat source, using an asymptotic model for the growth of the dry soil zone (compare to figure 12).

ASYMPTOTIC APPROXIMATION TEMPERATURE PROFILES

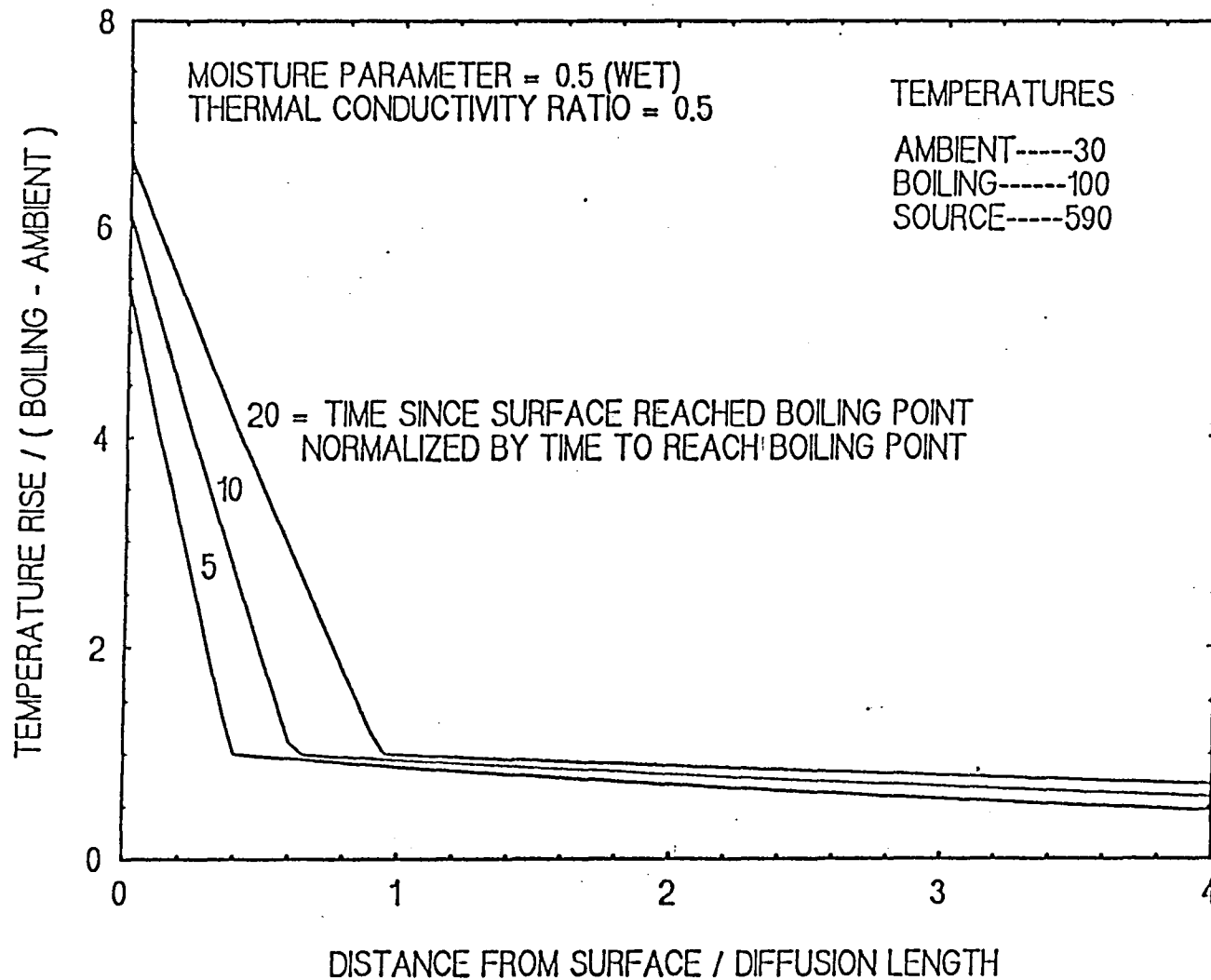


Fig. 22. Snapshot profiles of temperature rise as a function of distance into the soil, using an asymptotic model for the growth of the dry soil zone (compare to figure 13). The soil in this case is fully saturated sand.

ASYMPTOTIC APPROXIMATION TEMPERATURE PROFILES

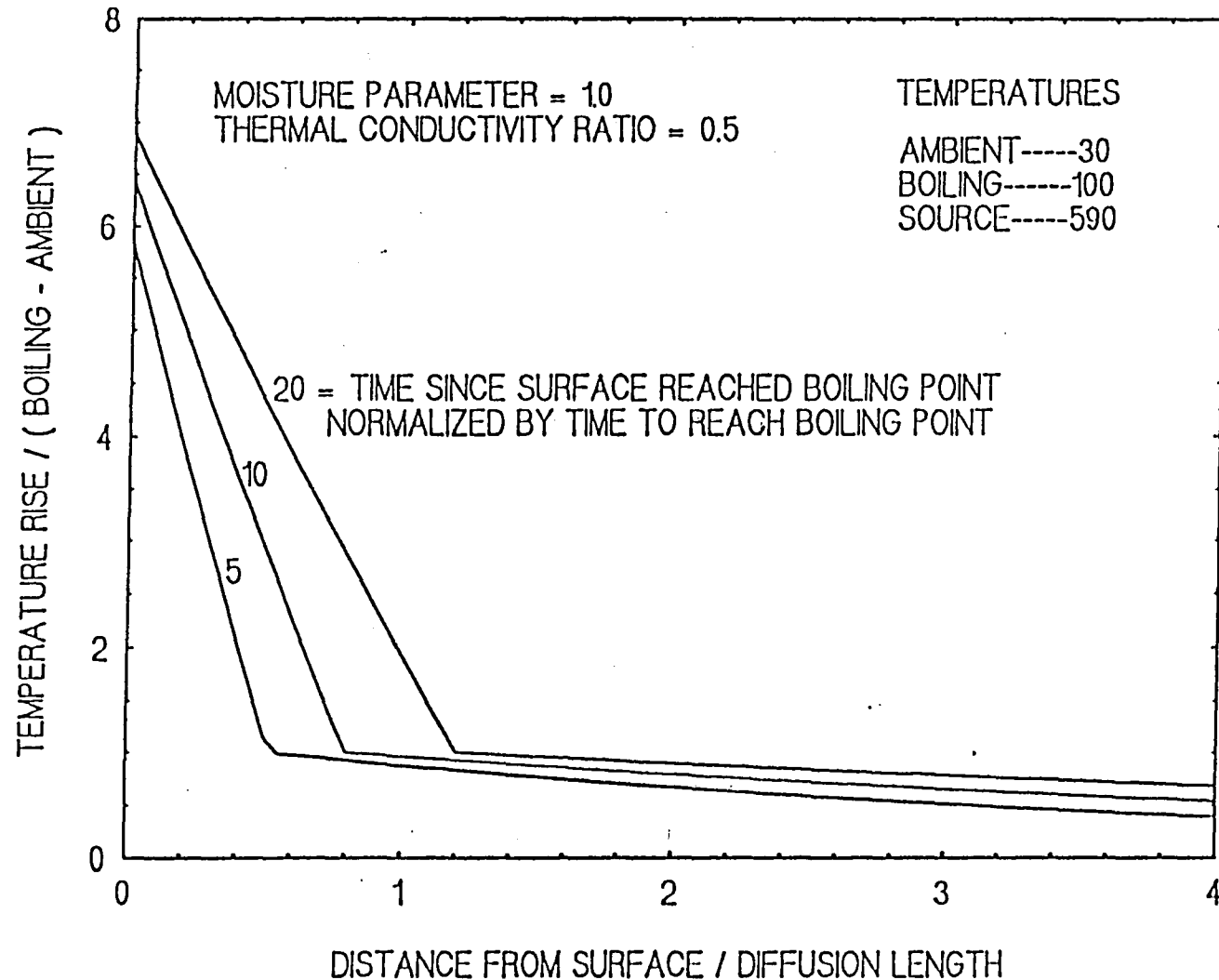


Fig. 23. Snapshot profiles of temperature rise as a function of distance into the soil, using an asymptotic model for the growth of the dry soil zone (compare to figure 14). The soil in this case is 50% saturated sand.

ASYMPTOTIC APPROXIMATION TEMPERATURE PROFILES

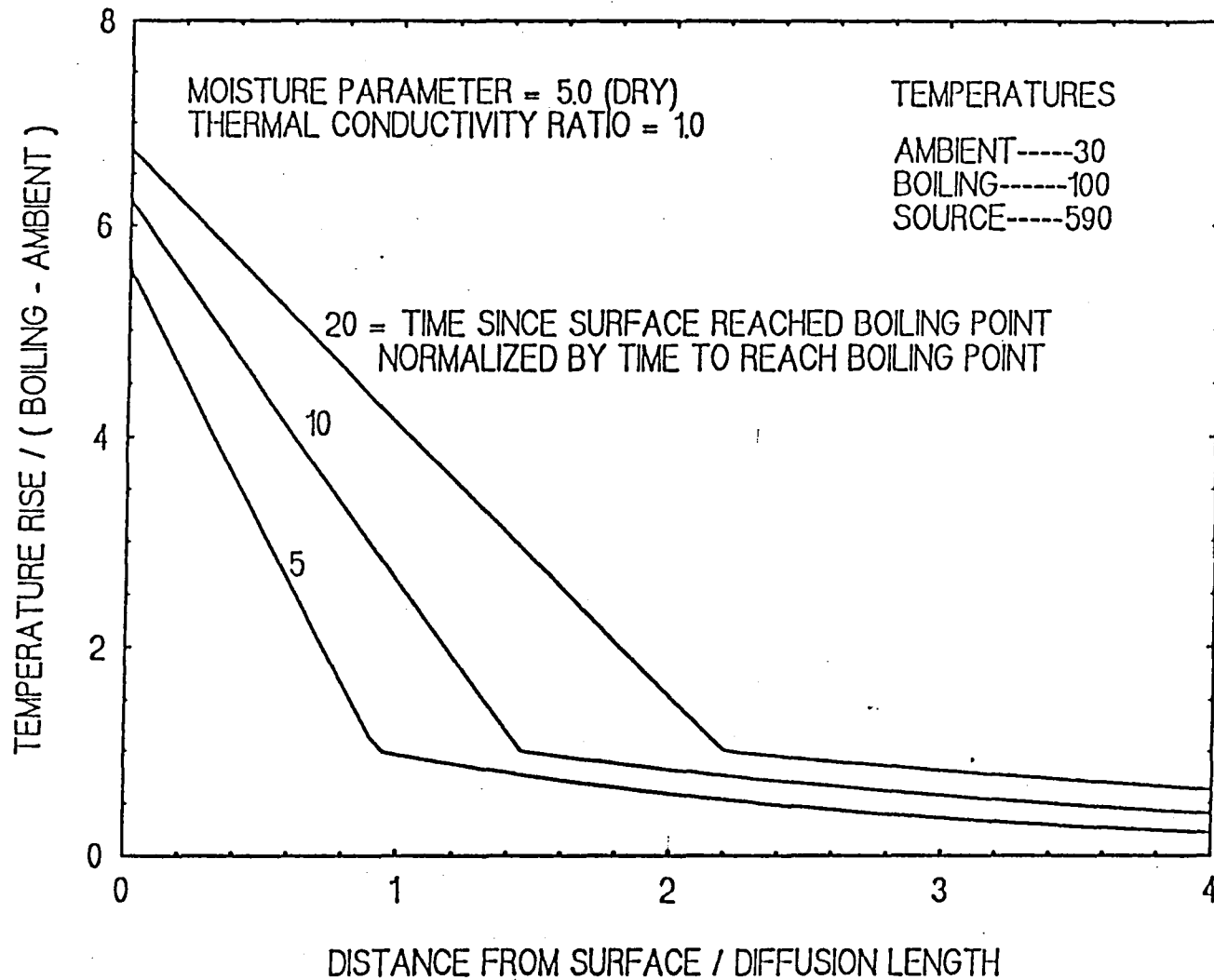
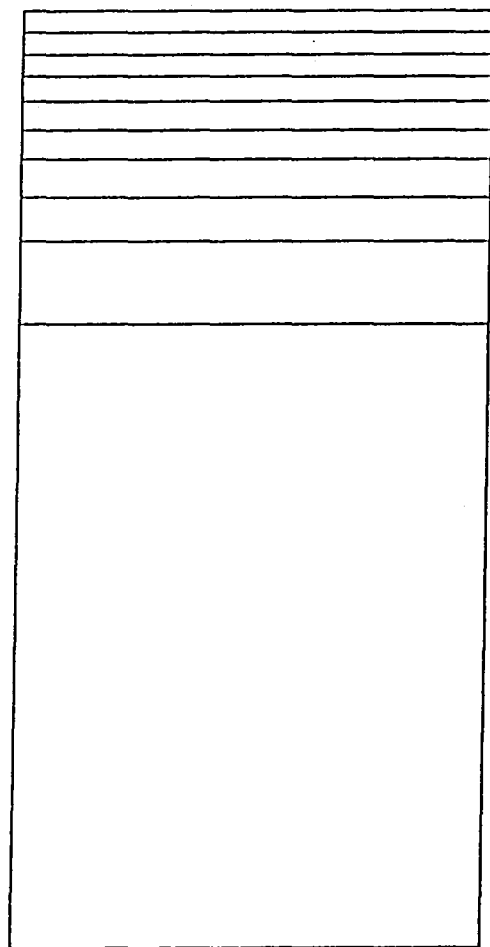
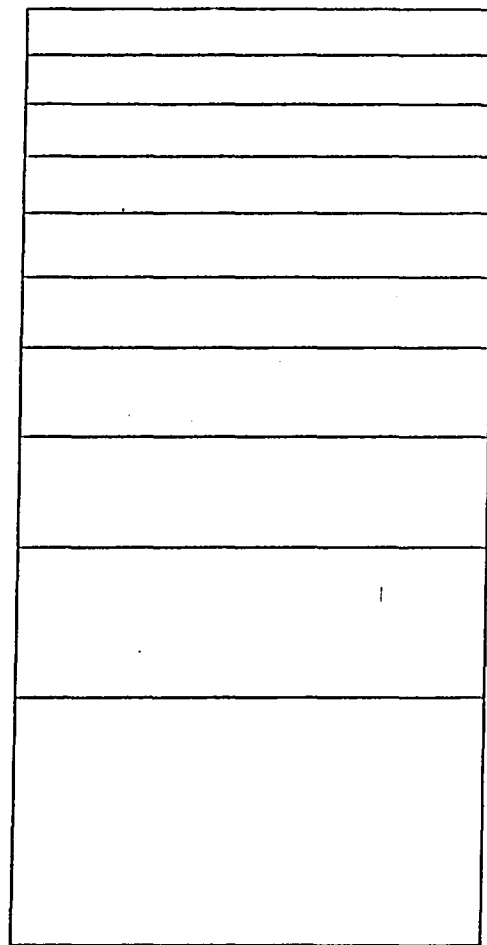


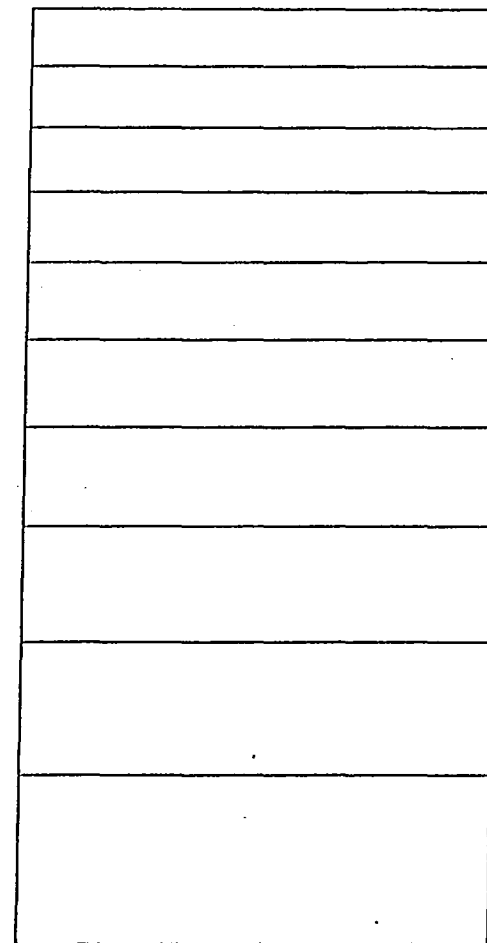
Fig. 24. Snapshot profiles of temperature rise as a function of distance into the soil, using an asymptotic model for the growth of the dry soil zone (compare to figure 15). The soil in this case is very dry sand.



(a)



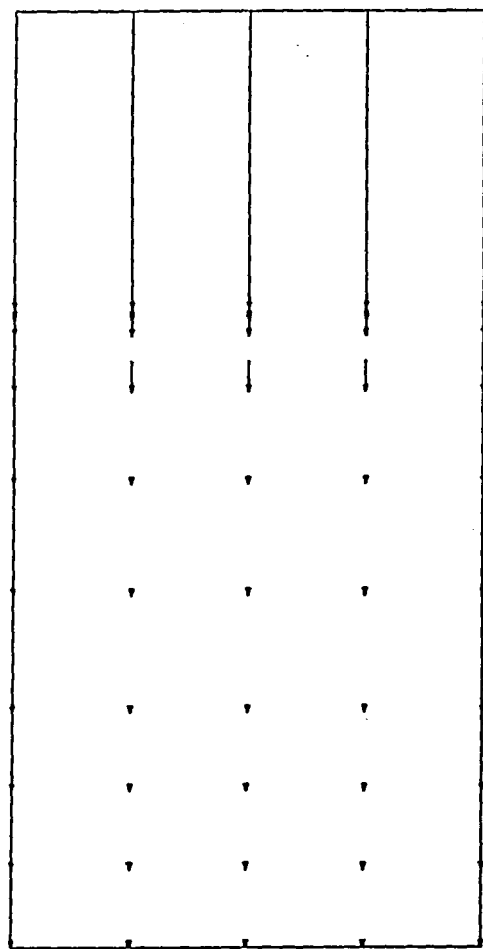
(b)



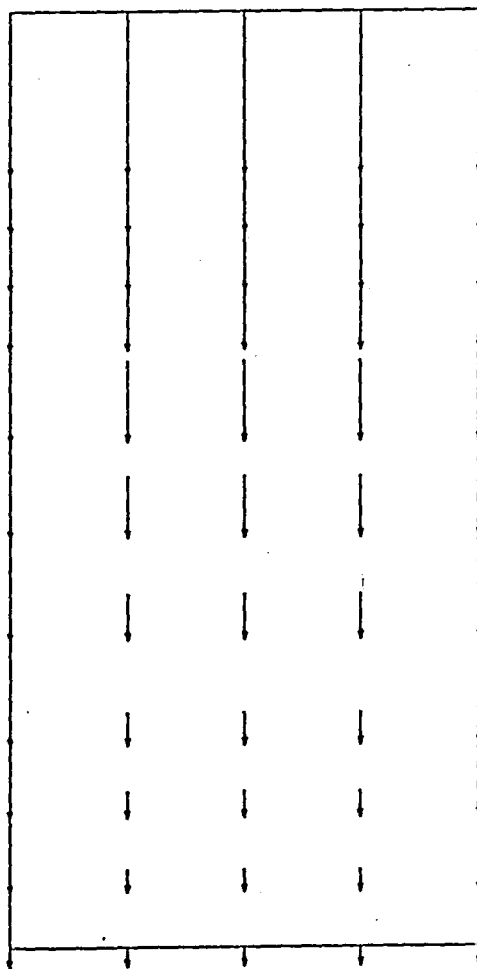
(c)

Fig. 25

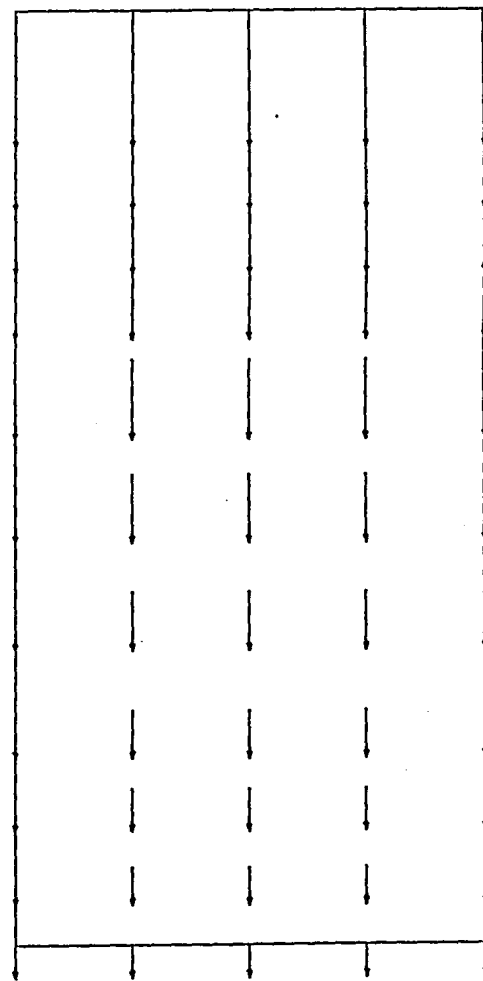
Isotherms for a medium at different times



(a)



(b)



(c)

Fig. 26

Heat flux vectors for a medium at different times

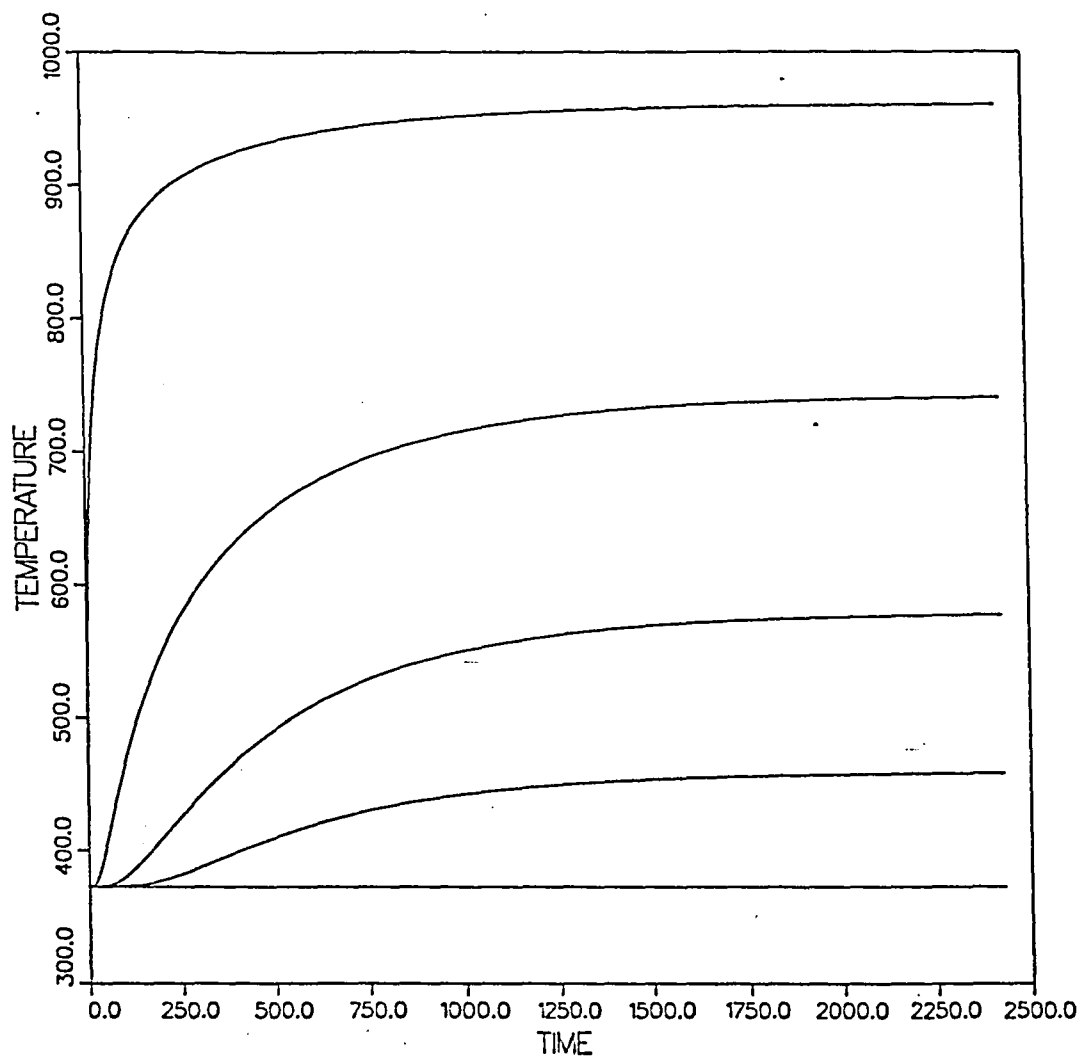
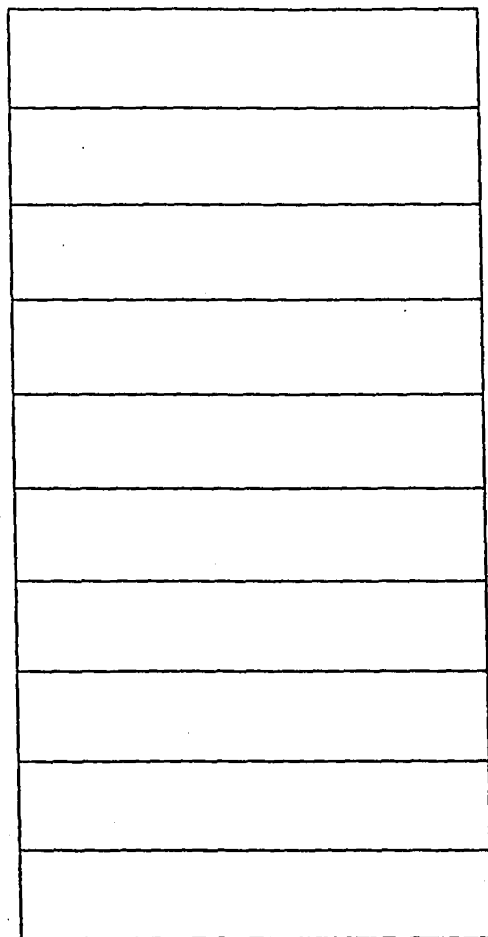
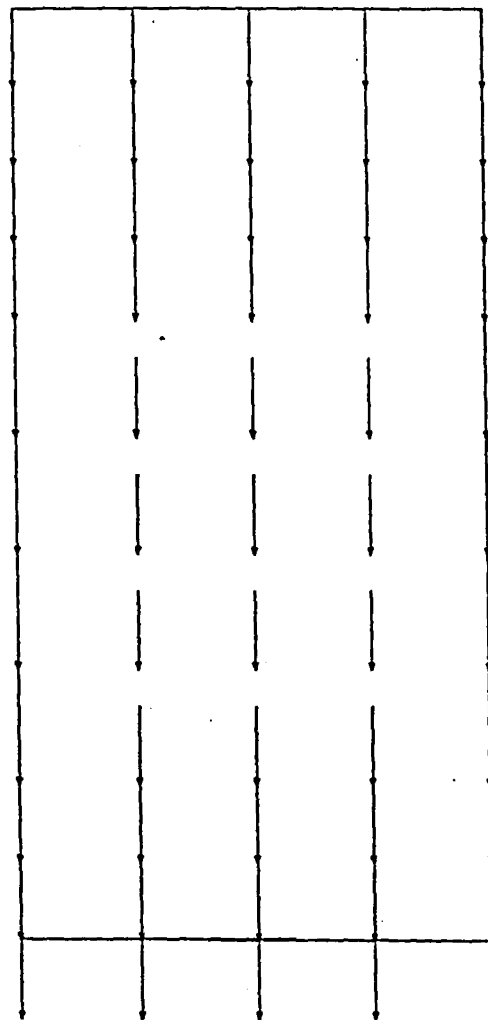


Fig. 27 Temperature histories at different depths



(a) Isotherms



(b) Heat flux vectors

Fig. 28 Steady state isotherms and heat flux vectors for a medium without any vapor mass influx

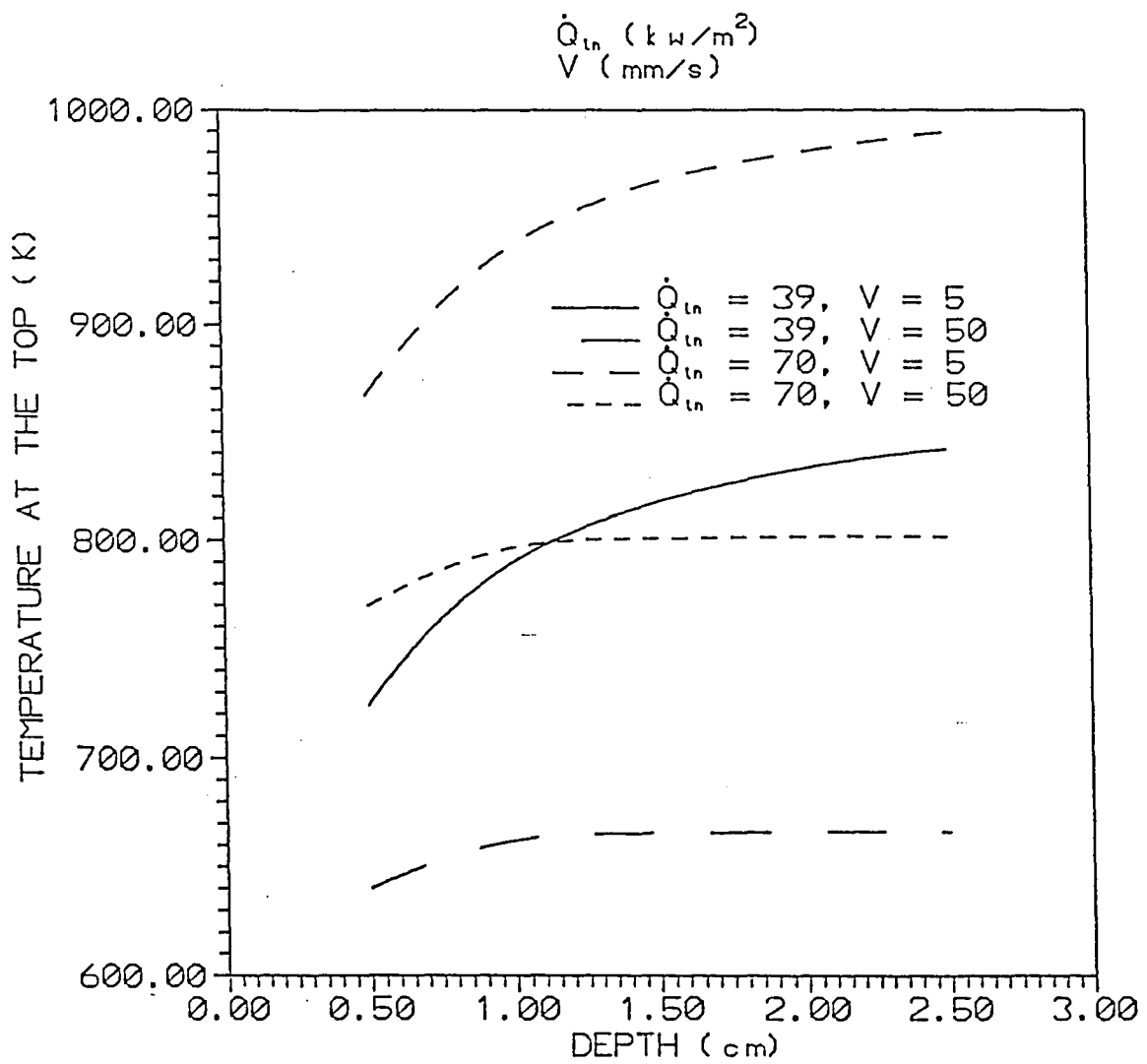


Fig. 29 Variation of temperature at the top surface (T_{top}) with depth; Quincy Sand

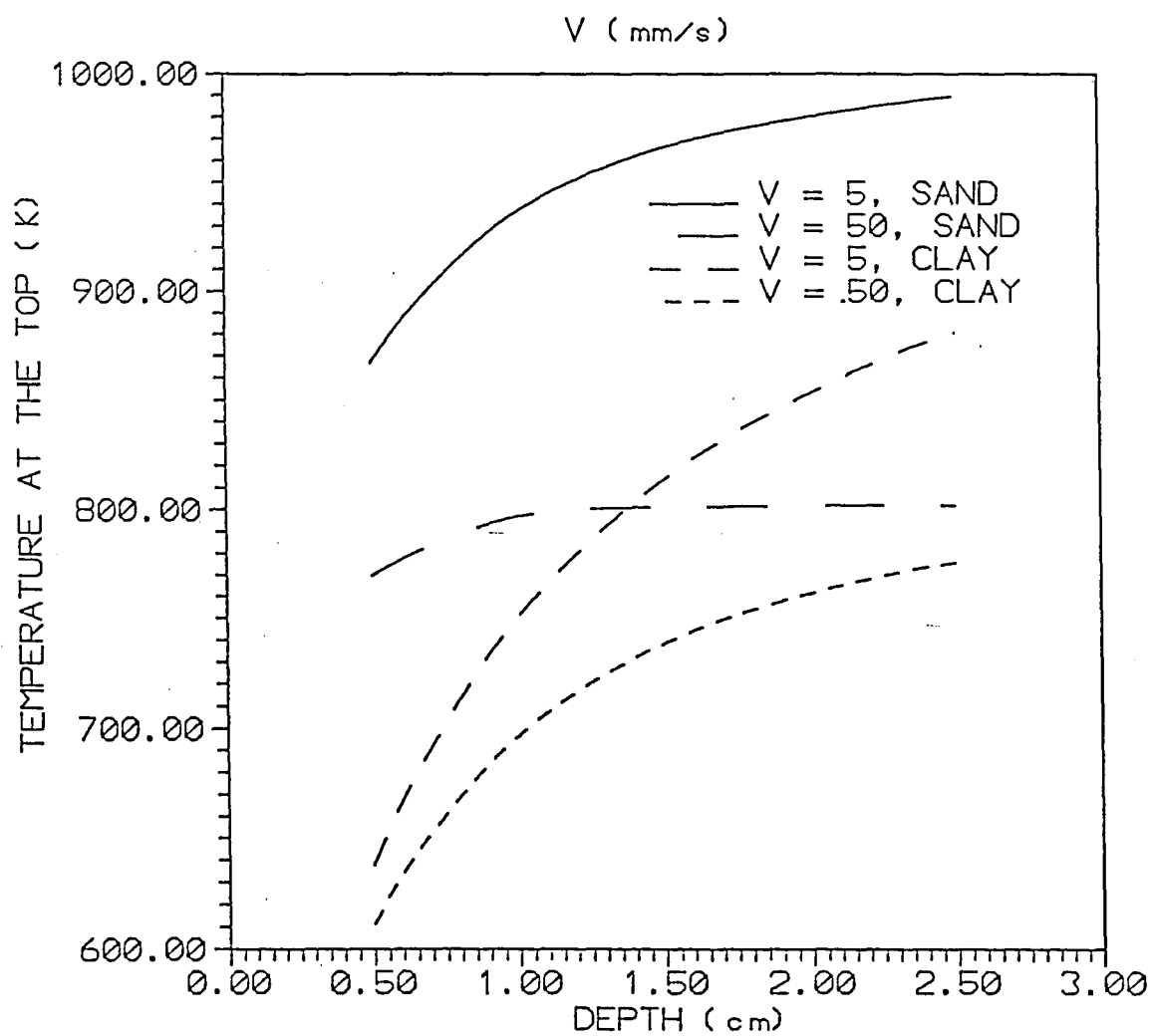


Fig. 30 Variation of temperature at the top surface (T_{top}) for different types of soils (sand and clay); $\dot{Q}_{in}'' = 70 \text{ kW/m}^2$

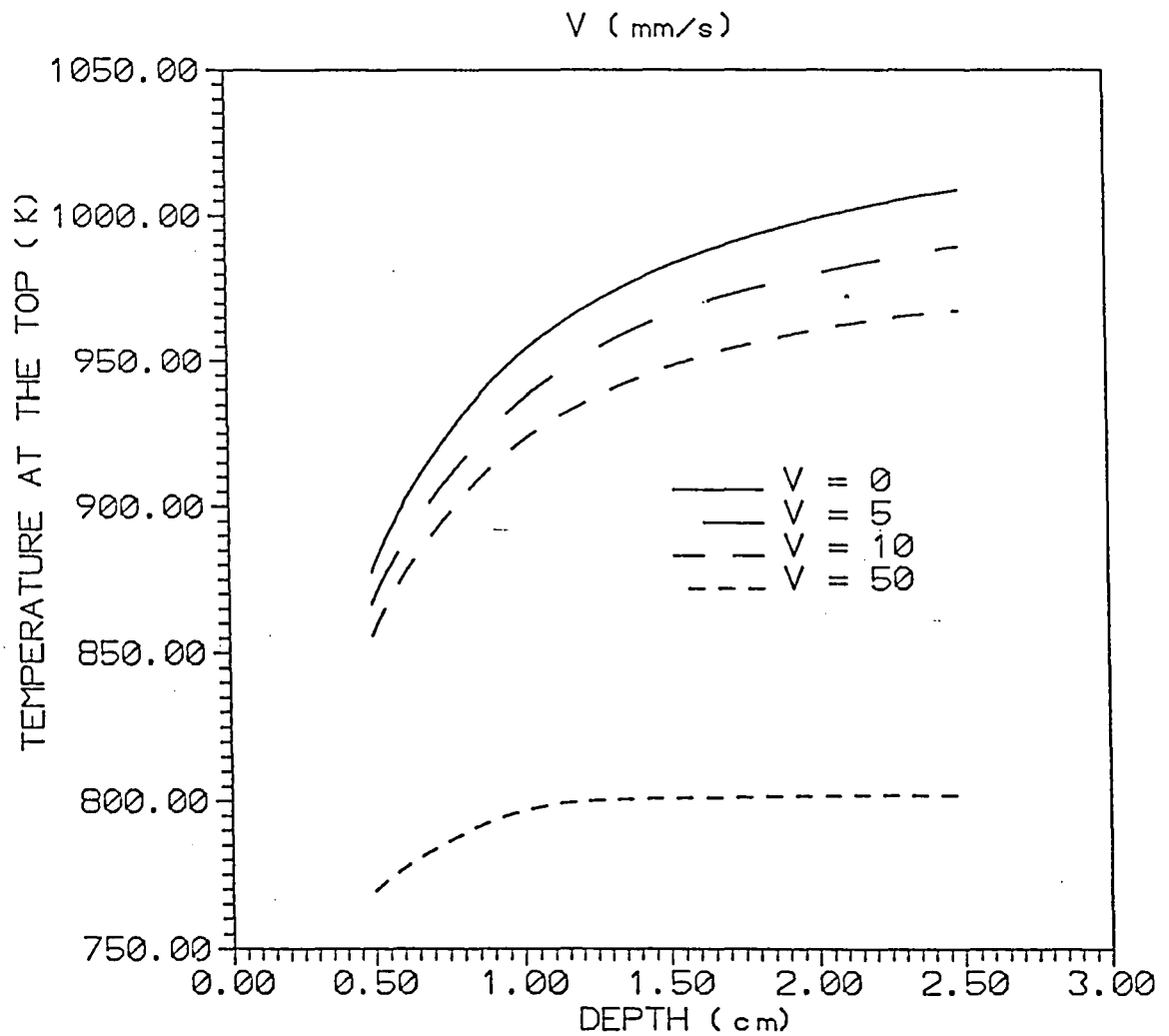


Fig. 31 The effect of vapor velocity (V) on the temperature at the top surface (T_{top}); Quincy Sand, $\dot{Q}_{in} = 70 \text{ kW/m}^2$

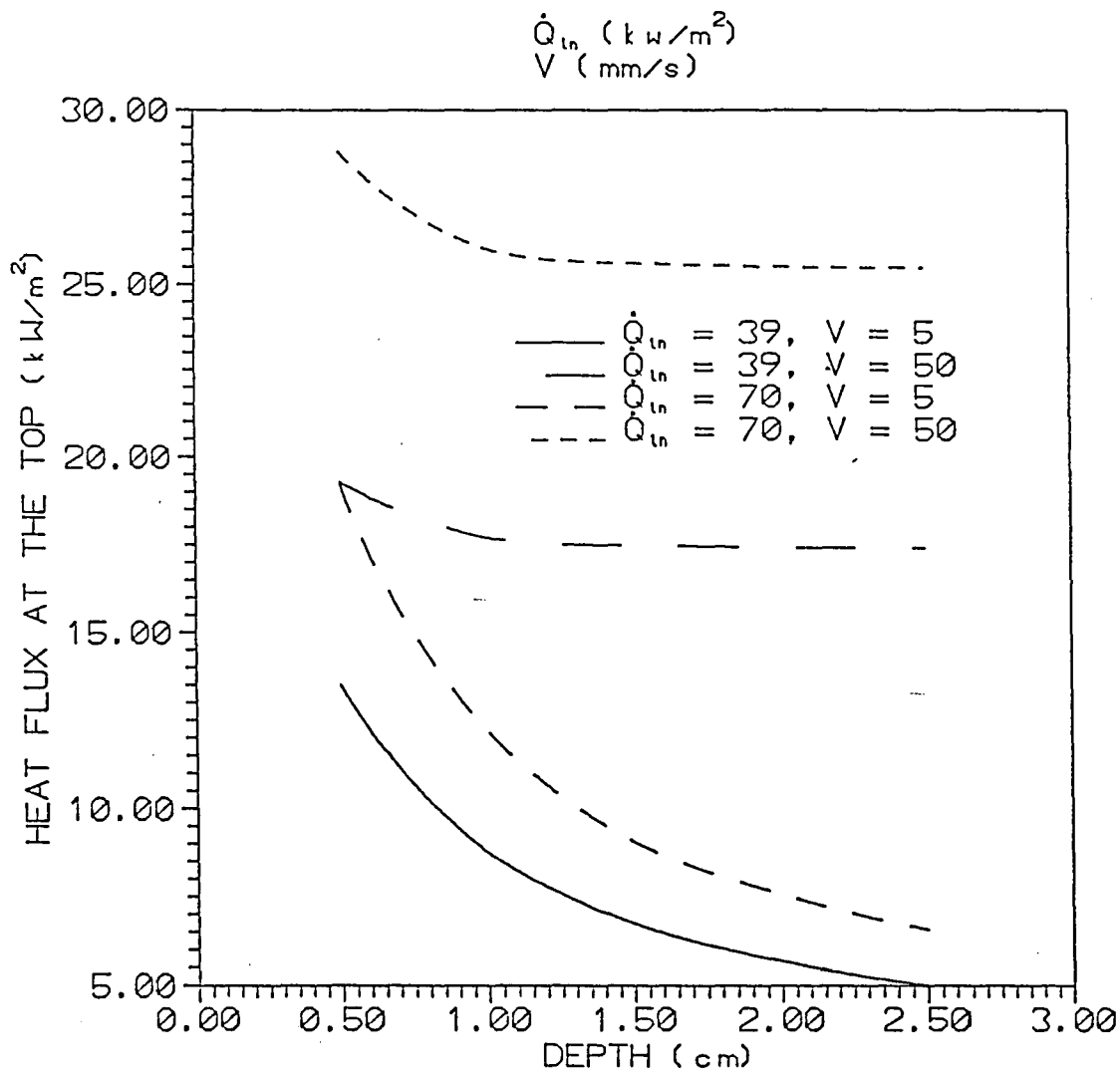


Fig. 32 Variation of heat flux at the top surface ($\dot{q}_{01}''$) with depth (d); Quincy Sand

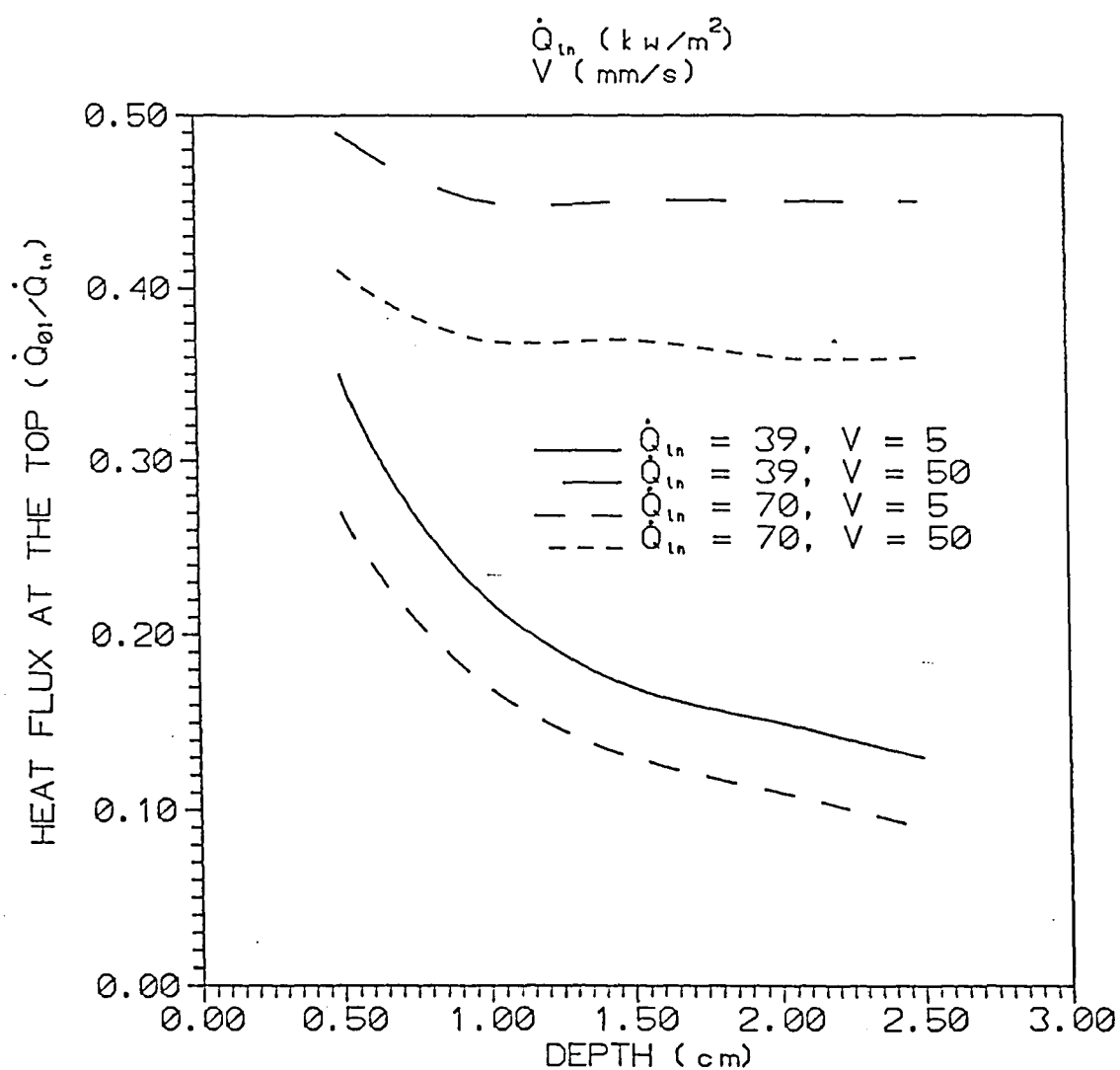


Fig. 33 Variation of nondimensional heat flux at top ($\dot{q}_{01}/\dot{Q}_{in}$) with depth; Quincy Sand

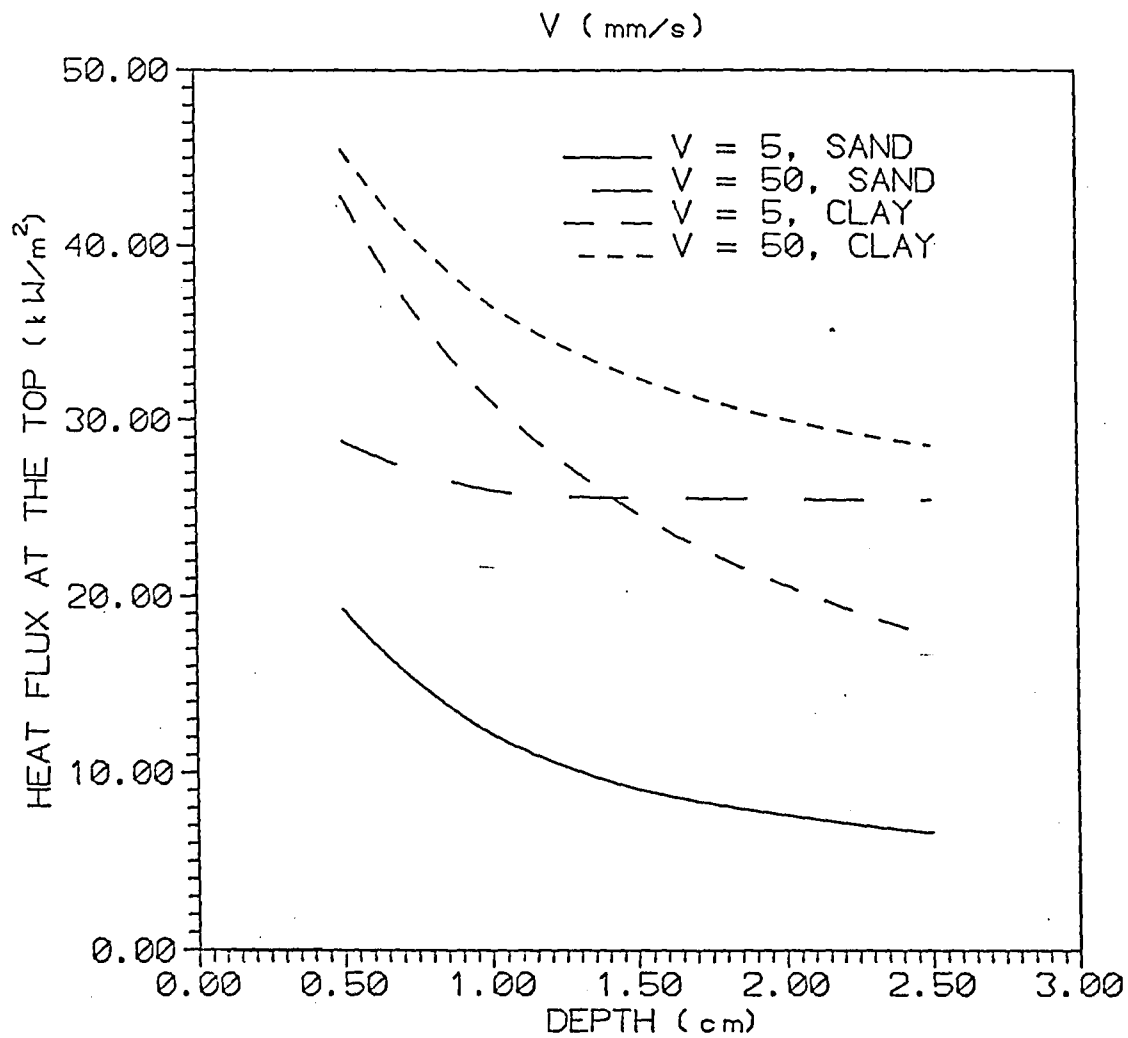


Fig. 34

Variation of heat flux at the top surface ($\dot{q}_{01}''$) for different types of soils (sand and clay); $\dot{Q}_{in}'' = 70 \text{ kW/m}^2$

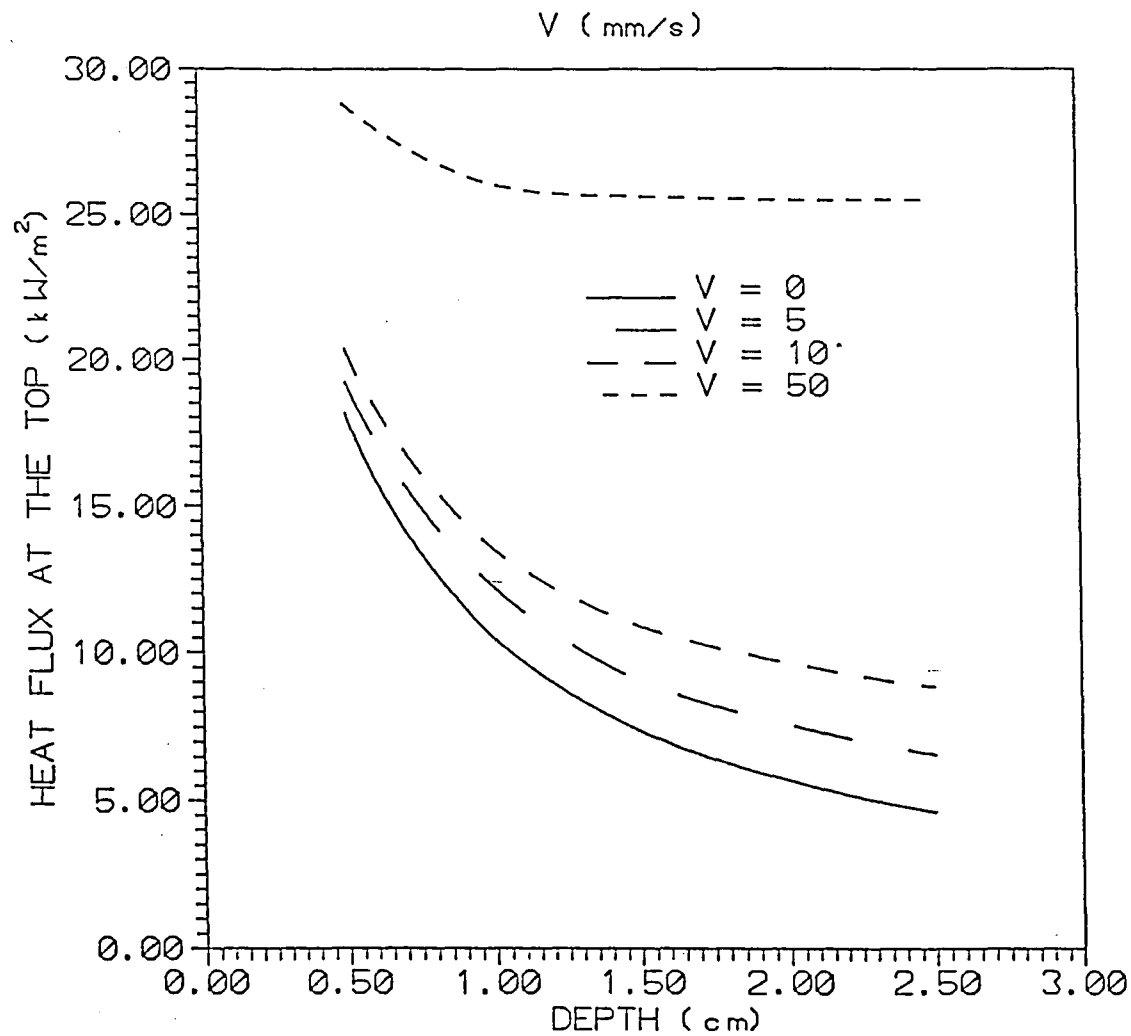


Fig. 35 The effect of vapor velocity (V) on the heat flux at the top surface ($\dot{q}_{01}''$); Quincy Sand, $\dot{Q}_{in}'' = 70 \text{ kW/m}^2$

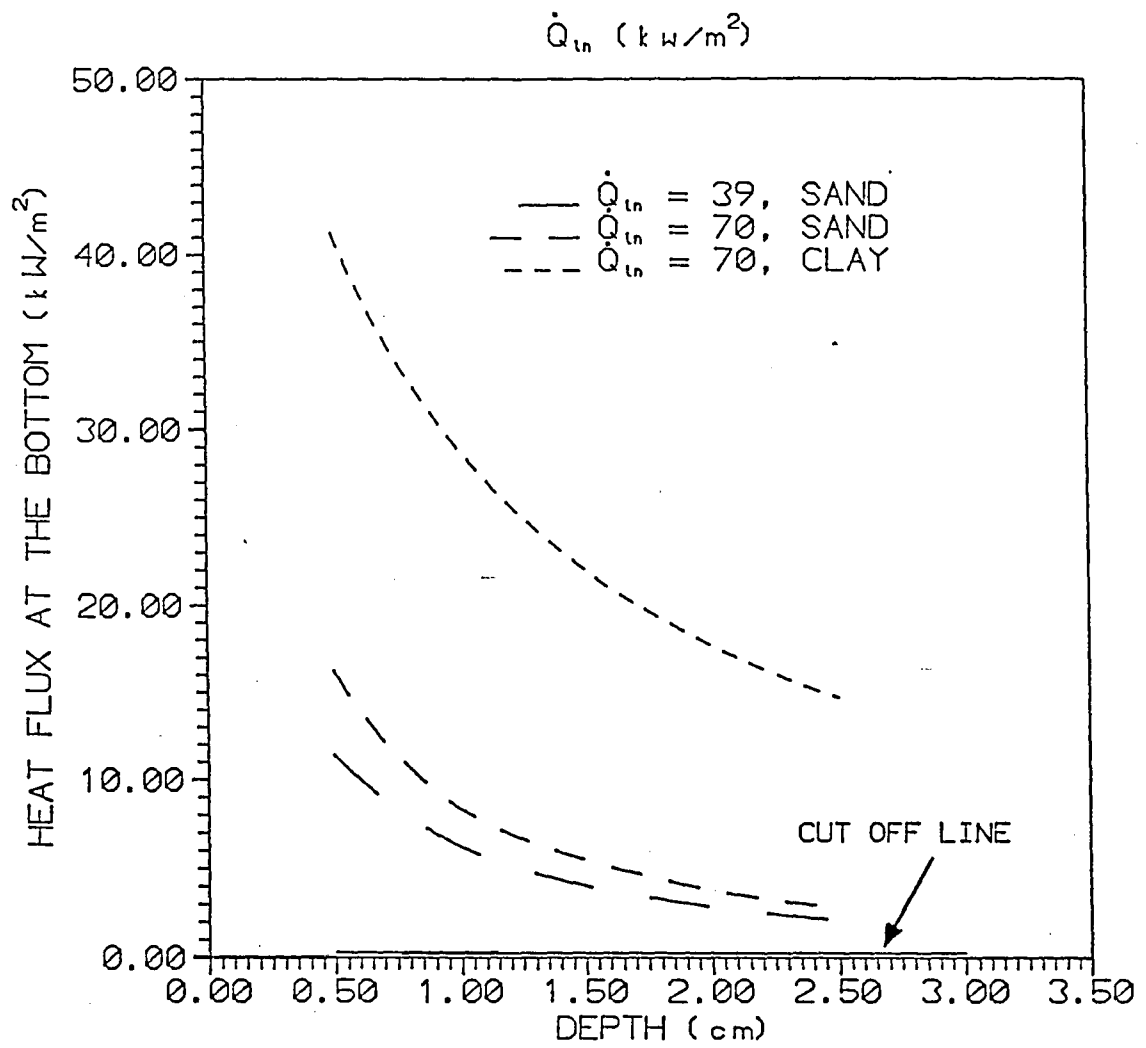


Fig. 36

Variation of heat flux from the dry zone to moist zone ($\dot{q}_{12}$) with depth for different types of soils (sand and clay); $V = 0.005 \text{ m/s}$

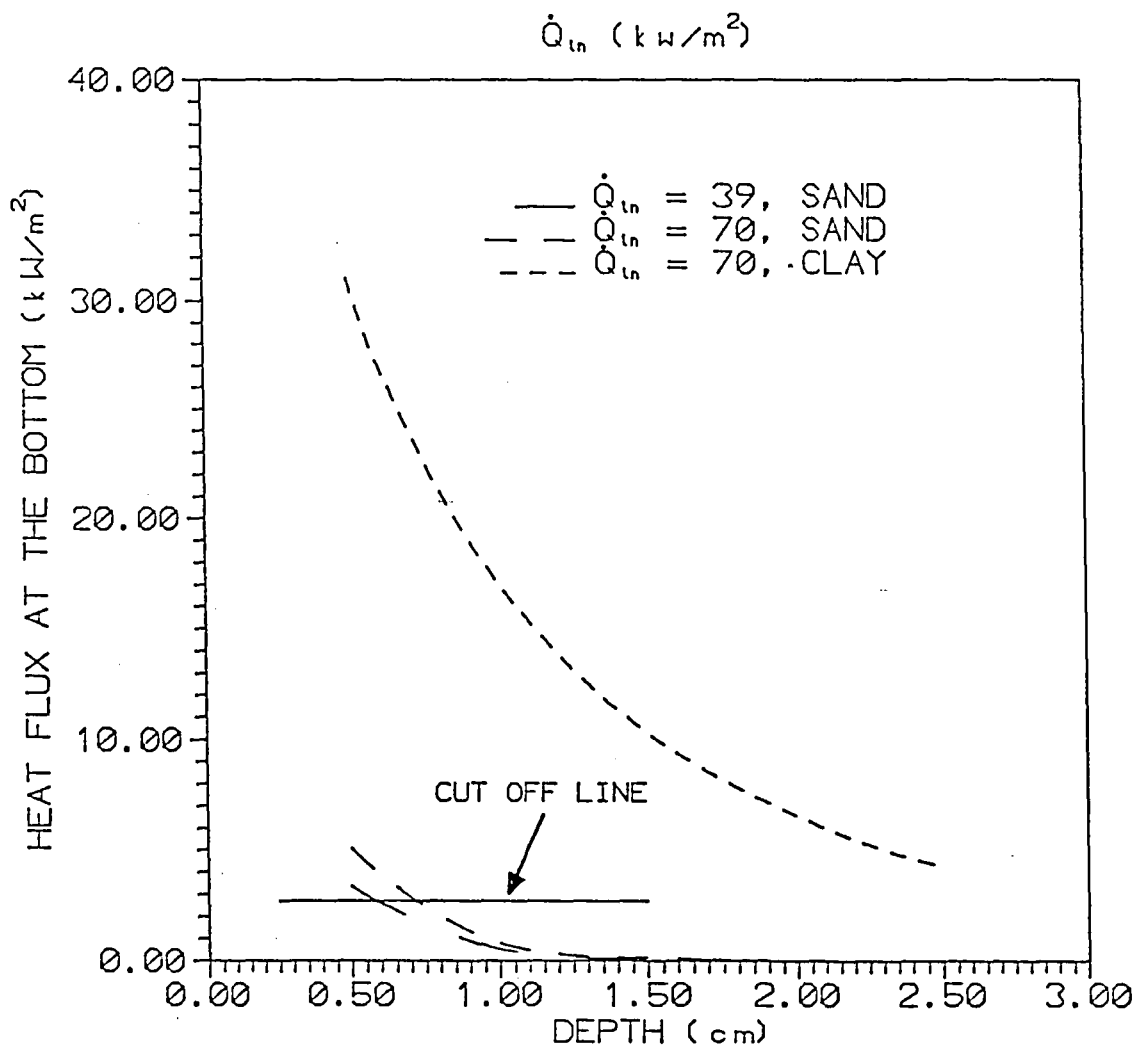


Fig. 37

Variation of heat flux from the dry zone to moist zone ($\dot{q}_{12}''$) with depth for different types of soils (sand and clay); $V = 0.05$ m/s

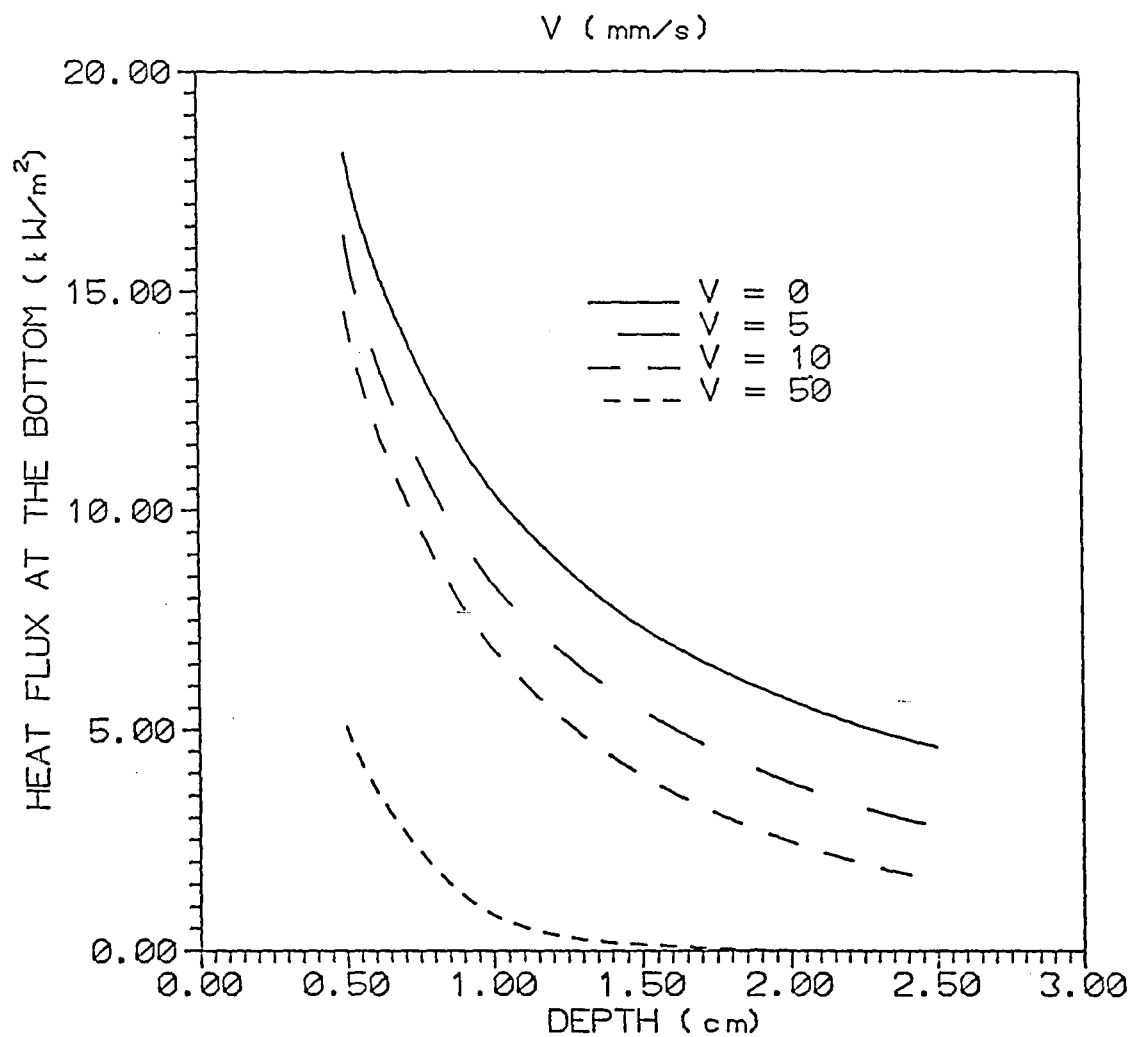


Fig. 38

The effect of vapor velocity (V) on the heat flux from the dry zone to moist zone ($\dot{q}_{12}''$); Quincy Sand, $\dot{Q}_{in}'' = 70 \text{ kW/m}^2$

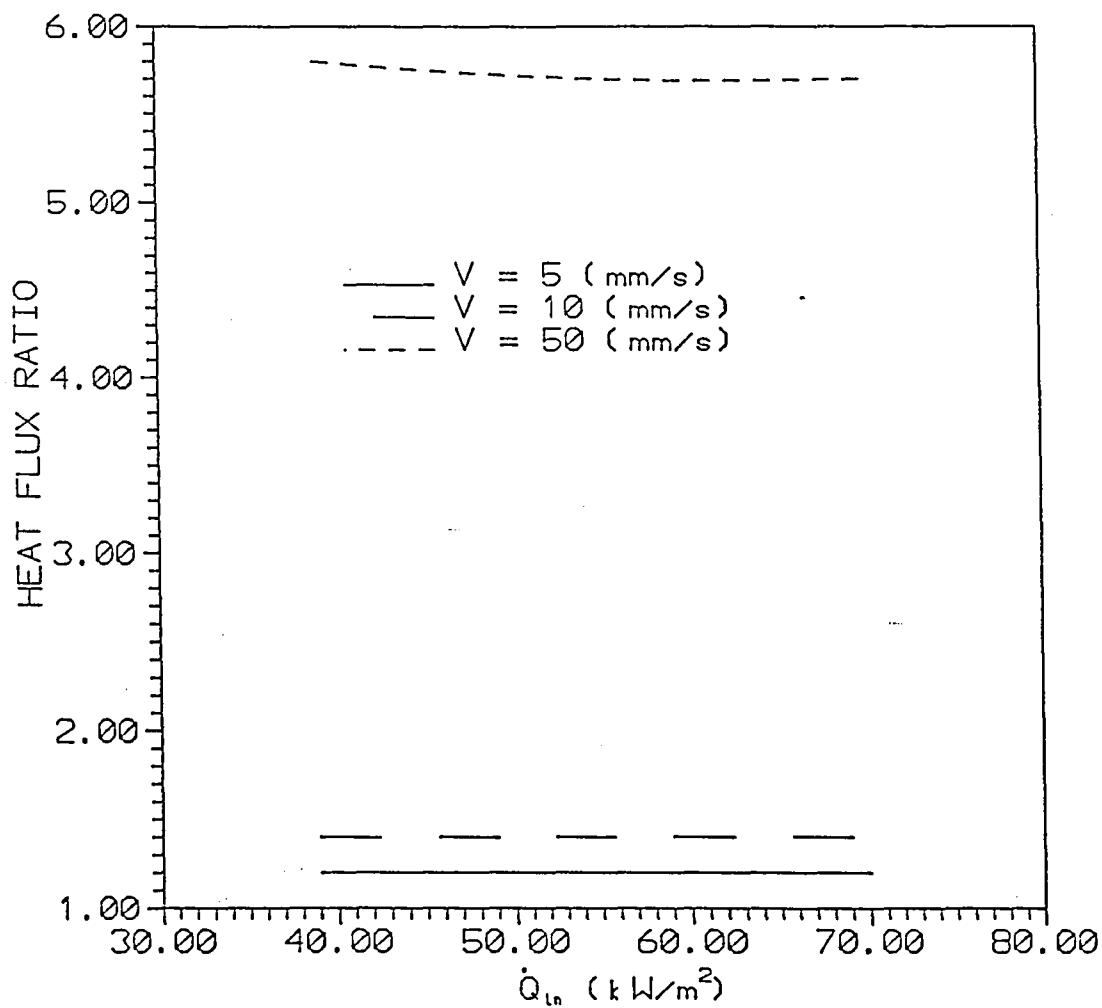


Fig. 39 Heat flux ratio ($\dot{q}_{01}/\dot{q}_{12}$) for a medium (sand) with dry zone depth, $d = 0.5 \text{ cm}$

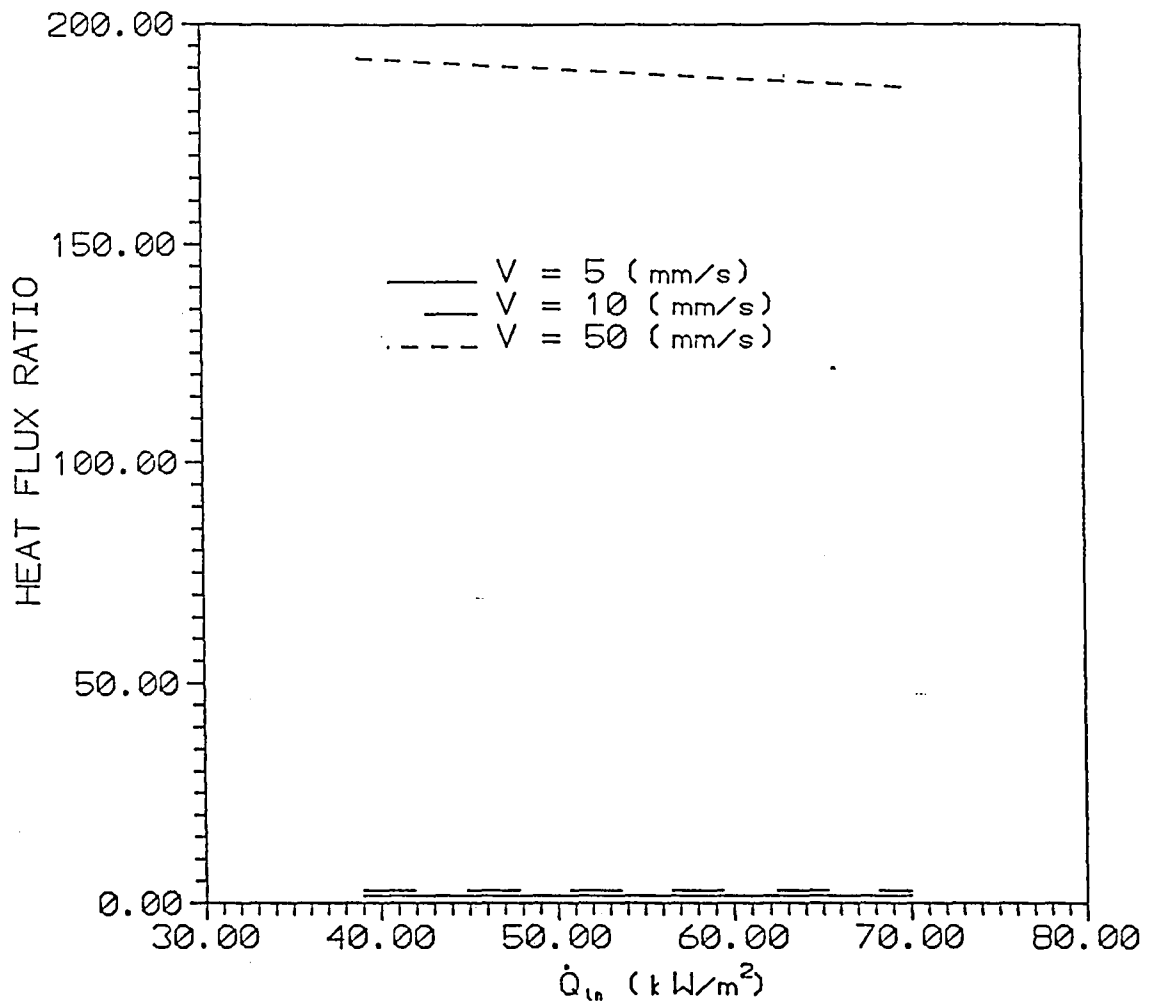


Fig. 40 Heat flux ratio ($\dot{q}_{01}/\dot{q}_{12}$) for a medium (sand) with dry zone depth, $d = 1.5$ cm